Muon collider ?

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Fermilab

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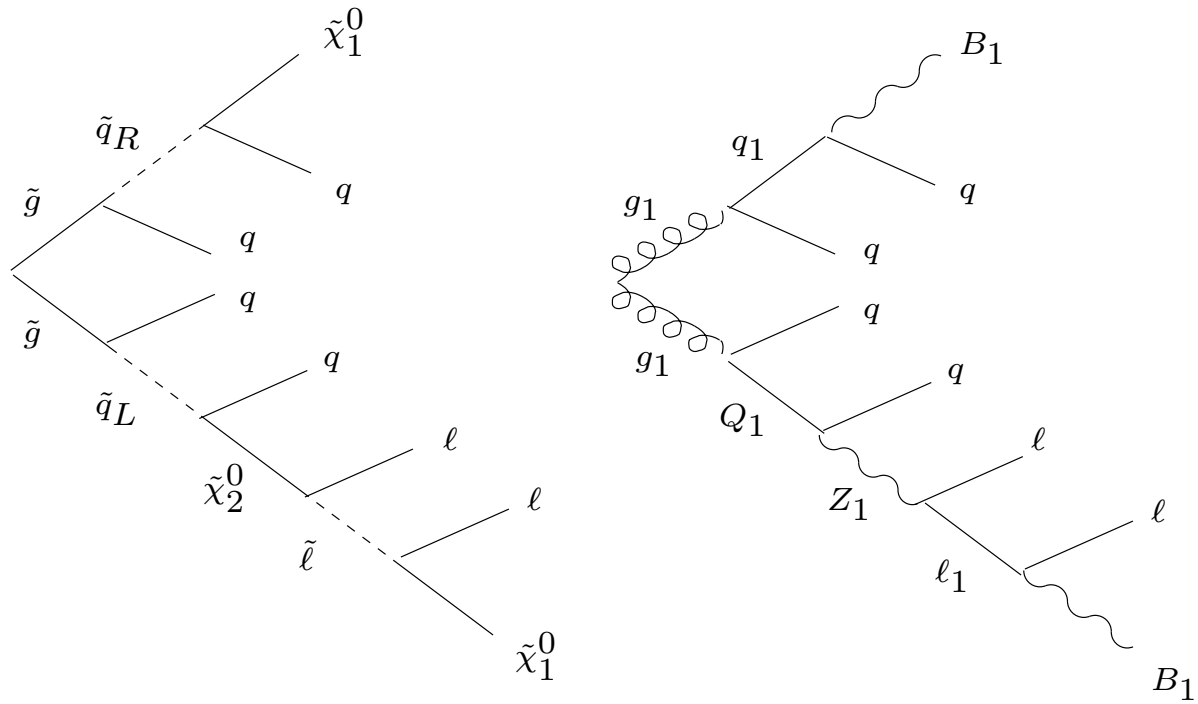
Plan

- A case study based on the CLIC report
  - Leptons and quarks are produced similarly
  - Slepton and KK-lepton pair production at 3 TeV CLIC
- Some of the basic questions
  - Effect of forward/backward lockout angle
  - Luminosity?
  - Polarization?
  - Beamstrahlung?

Why muon collider?

- Why lepton collider?
  - Many of new physics signatures look very similar at hadron colliders
  - Supersymmetry, extra dimensions, little higgs model and so on
  - The identity of discovery at the LHC is unclear (we will have to wait and see)
  - Need to measure the masses/spins/interactions(couplings) of new particles
    $\longrightarrow$ lepton collider is the best (discrimination and precision measurement)
- Why high energy?
  - $\sigma(\ell^+\ell^- \longrightarrow W^+W^-\nu\bar{\nu} \longrightarrow \nu\bar{\nu}h) \sim \log\left(\frac{s}{m_h^2}\right)$
  - ILC people talk about low-lying supersymmetric states only
  - There are many SUSY models that have heavy particles
    $i.e.$, focus point SUSY, compressed SUSY and so on
  - Large CM energy will cover whole mass spectrum
    $\longrightarrow$ important to understand SUSY breaking mechanism
  - Some of extra dimensional models have larger mass scale
    $\longrightarrow$ 1 TeV ILC may not see anything
- Why muon collider?
  - Any scalar particle that has a Yukawa type interaction will have a better chance to be seen at a muon collider
  - Better to have an option to go for higher energies starting from 1 TeV

Typical event in SUSY and UED

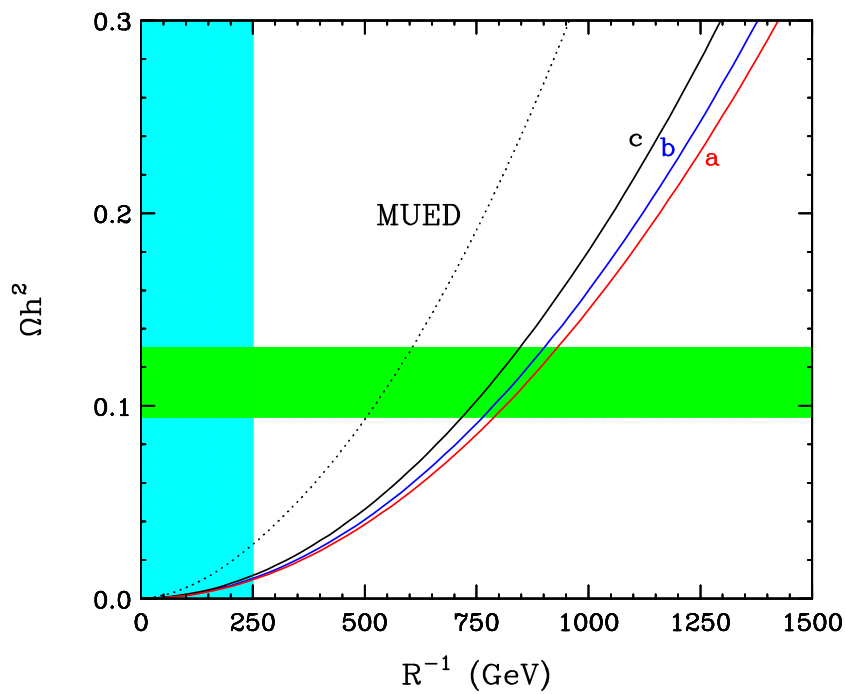


- Both have similar diagrams $\longrightarrow$ same signatures!
  - At first sight, it is not clear which model we are considering
- A lot of jets $\longrightarrow$ correct jet identification is difficult $\longrightarrow$ ISR/FSR add more confusion

Why high energy?

(Kong, Matchev, hep-ph/0509119)

- Cosmology requires large LKP (lightest KK particle) mass unlike SUSY.
- Preferred LKP mass is around 500-600 GeV (for $0.094 < \Omega_{CDM}h^2 < 0.129$). However this depends on the model. Generally LKP mass can be as high as a few TeV. $\longrightarrow$ 1 TeV ILC may not discover UED. $\longrightarrow$Need larger energy. $\longrightarrow$ 3 TeV lepton collider will be good enough to see first KK mode.



Why high energy?

(Berger, Gainer, Hewett, Lillie, Rizzo, arXiv:0711.1374)

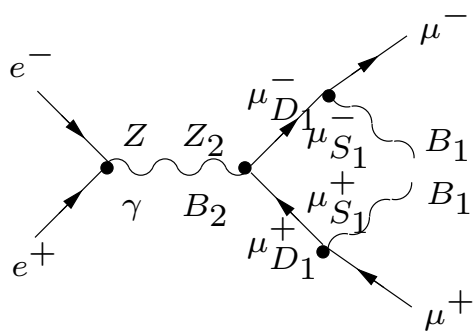
- We address the question whether the ILC can resolve the LHC Inverse Problem within the framework of the MSSM. We examine 242 points in the MSSM parameter space which were generated at random and were found to give indistinguishable signatures at the LHC. After a realistic simulation including full Standard Model backgrounds and a fast detector simulation, we find that roughly only one third of these scenarios lead to visible signatures of some kind with a significance ≥ 5 at the ILC with $\sqrt{s} = 500$ GeV. Furthermore, we examine these points in parameter space pairwise and find that only one third of the pairs are distinguishable at the ILC at 5σ.
- Larger luminosity may help (250 fb^{-1} was used in the above analysis)
- It is more important to cover more particles $\longrightarrow$ need higher energy

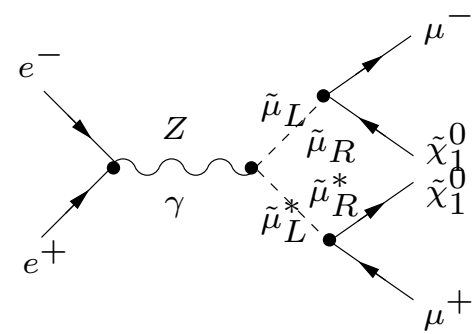
Supersymmetric spectra in post-WMAP benchmarks calculated with `ISASUGRA 7.67`

Model	A'	B'	C'	D'	E'	F'	G'	H'	I'	J'	K'	L'	M'
$m_{1/2}$	600	250	400	525	300	1000	375	935	350	750	1300	450	1840
m_0	107	57	80	101	1532	3440	113	244	181	299	1001	303	1125
$\tan\beta$	5	10	10	10	10	10	20	20	35	35	46	47	51
sign(μ)	$+$	$+$	$+$	$-$	$+$	$+$	$+$	$+$	$+$	$+$	$-$	$+$	$+$
m_t	175	175	175	175	171	171	175	175	175	175	175	175	175
Masses													
$\lvert\mu(m_Z)\rvert$	773	339	519	663	217	606	485	1092	452	891	1420	563	1940
h	116	113	117	117	114	118	117	122	117	121	123	118	124
H	896	376	584	750	1544	3525	525	1214	444	888	1161	480	1623
A	889	373	580	745	1534	3502	522	1206	441	882	1153	477	1613
$H^{\pm}$	899	384	589	754	1546	3524	532	1217	453	892	1164	490	1627
χ	242	95	158	212	112	421	148	388	138	309	554	181	794
χ_2	471	180	305	415	184	610	286	750	266	598	1064	351	1513
χ_3	778	345	525	671	229	622	492	1100	459	899	1430	568	1952
χ_4	792	366	540	678	302	858	507	1109	475	908	1437	582	1959
$\chi_1^{\pm}$	469	178	304	415	175	613	285	750	265	598	1064	350	1514
$\chi_2^{\pm}$	791	366	541	679	304	846	507	1108	475	908	1435	582	1956
$\tilde{g}$	1367	611	940	1208	800	2364	887	2061	835	1680	2820	1055	3884
e_L, μ_L	425	188	290	376	1543	3499	285	679	304	591	1324	434	1660
e_R, μ_R	251	117	174	224	1534	3454	185	426	227	410	1109	348	1312
ν_e, ν_μ	412	167	274	362	1539	3492	270	665	290	579	1315	423	1648
τ_1	249	109	167	217	1521	3427	157	391	150	312	896	194	796
τ_2	425	191	291	376	1534	3485	290	674	312	579	1251	420	1504
ν_τ	411	167	273	360	1532	3478	266	657	278	558	1239	387	1492
u_L, c_L	1248	558	859	1103	1639	3923	814	1885	778	1554	2722	1001	3670
u_R, c_R	1202	542	830	1064	1637	3897	787	1812	754	1497	2627	969	3528
d_L, s_L	1250	564	863	1107	1641	3924	818	1887	783	1556	2723	1004	3671
d_R, s_R	1197	541	828	1059	1638	3894	786	1804	752	1491	2615	967	3509
t_1	958	411	653	860	1052	2647	617	1477	584	1207	2095	753	2857
t_2	1184	576	837	1048	1387	3373	792	1753	748	1428	2366	920	3231
b_1	1147	514	789	1015	1375	3356	737	1719	677	1377	2297	844	3149
b_2	1181	535	816	1043	1602	3816	770	1761	725	1423	2349	904	3217

Proposed post-WMAP CMSSM benchmark points and mass spectra (in GeV), as calculated using `ISASUGRA 7.67`.

SUSY vs UED at LC in $\mu^+\mu^- + \not{E}_T$ channel





- Angular distribution

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{UED} \sim 1 + \frac{E^2_{\mu_1} - M^2_{\mu_1}}{E^2_{\mu_1} + M^2_{\mu_1}} \cos^2\theta \qquad \left(\frac{d\sigma}{d\cos\theta}\right)_{SUSY} \sim 1 - \cos^2\theta$$
$$\sim 1 + \cos^2\theta$$

- μ^- energy distribution

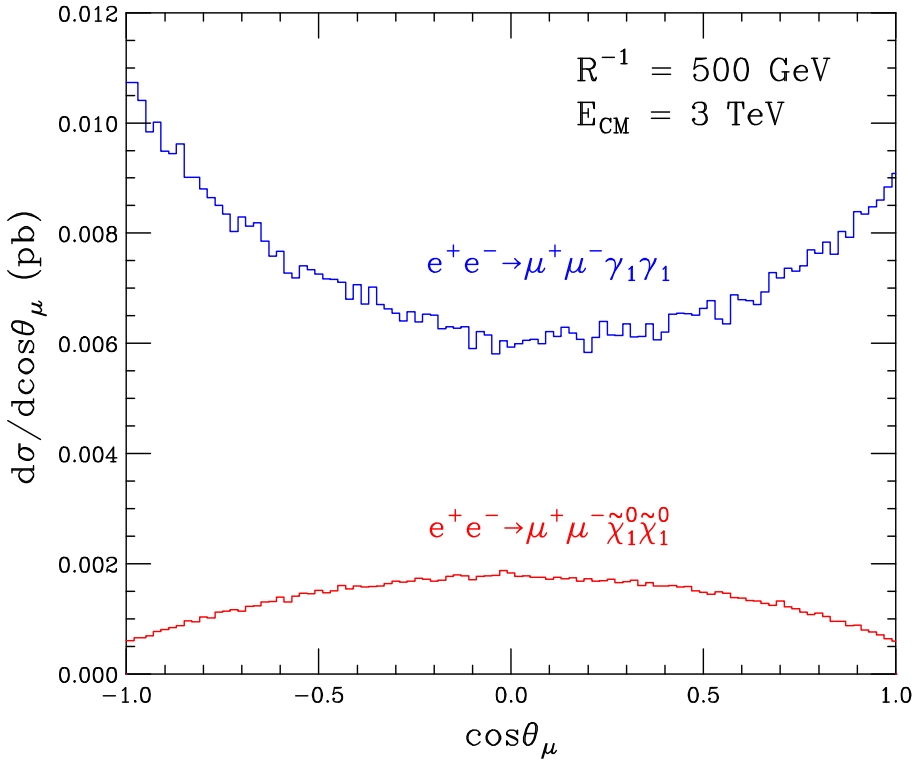
- Threshold scan

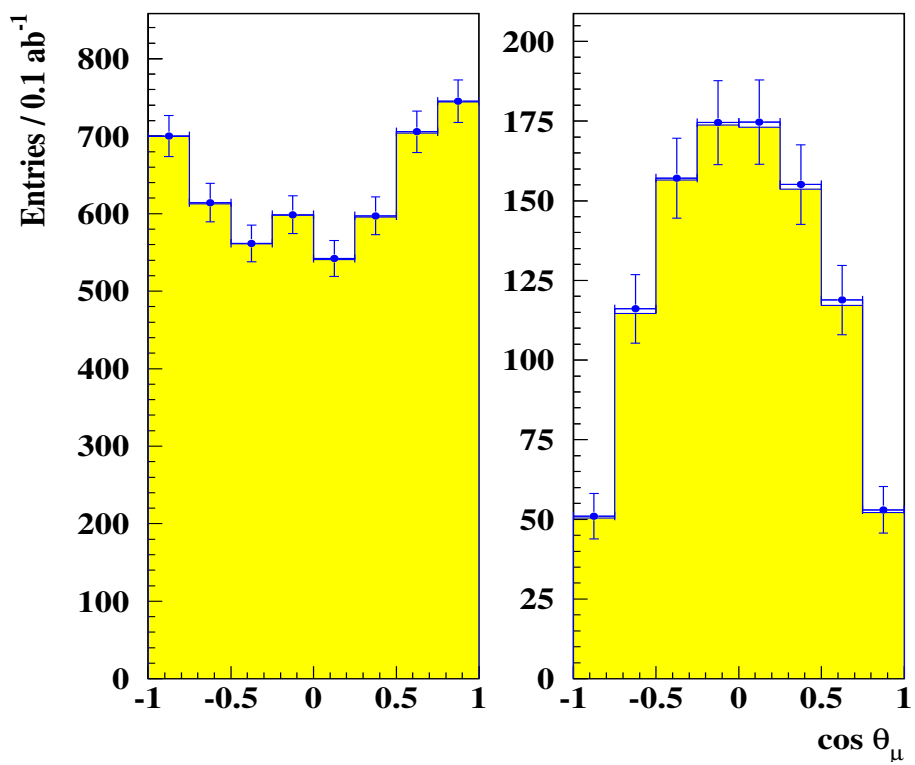
- Photon energy distribution

- Softwares : CompHEP/Pythia/SimDet/GuineaPig+parametrization

The Angular Distribution (LC)

(Battaglia, Datta, De Roeck, Kong, Matchev,hep-ph/0502041)

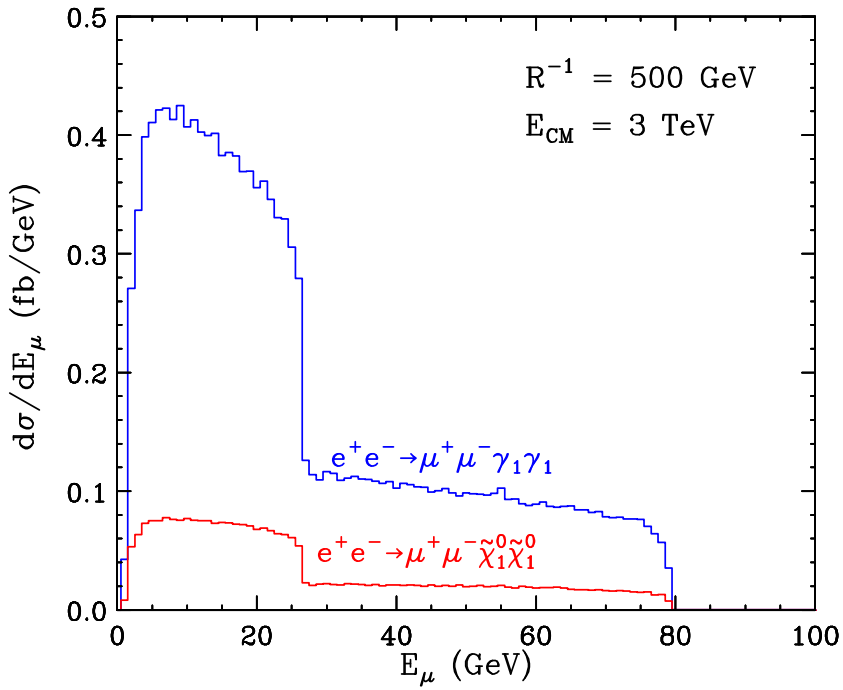


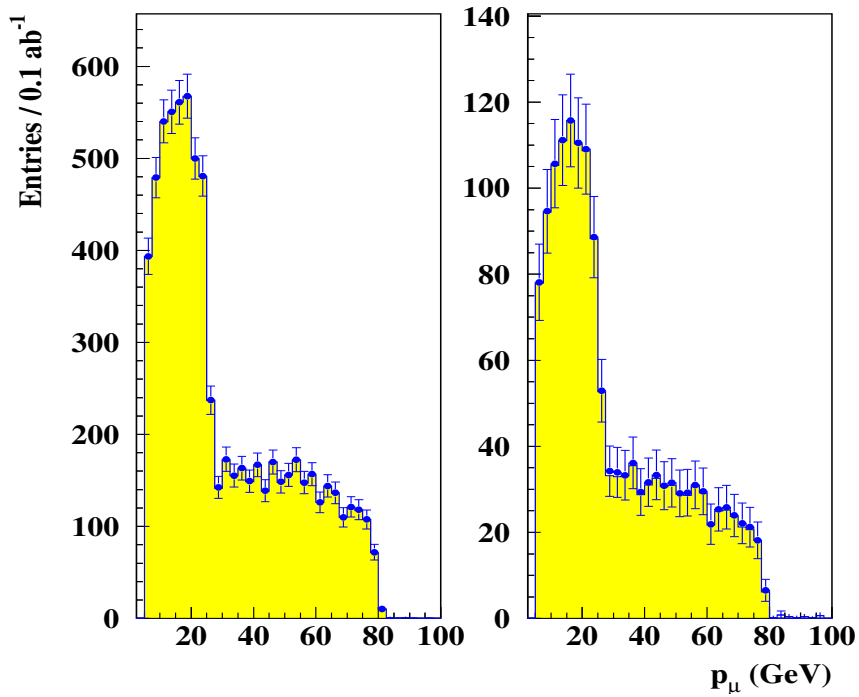


- $\left(\frac{d\sigma}{d\cos\theta}\right)_{UED} \sim 1 + \cos^2\theta$
- $\left(\frac{d\sigma}{d\cos\theta}\right)_{SUSY} \sim 1 - \cos^2\theta$

The μ Energy Distribution (LC)

(Battaglia, Datta, De Roeck, Kong, Matchev,hep-ph/0502041)

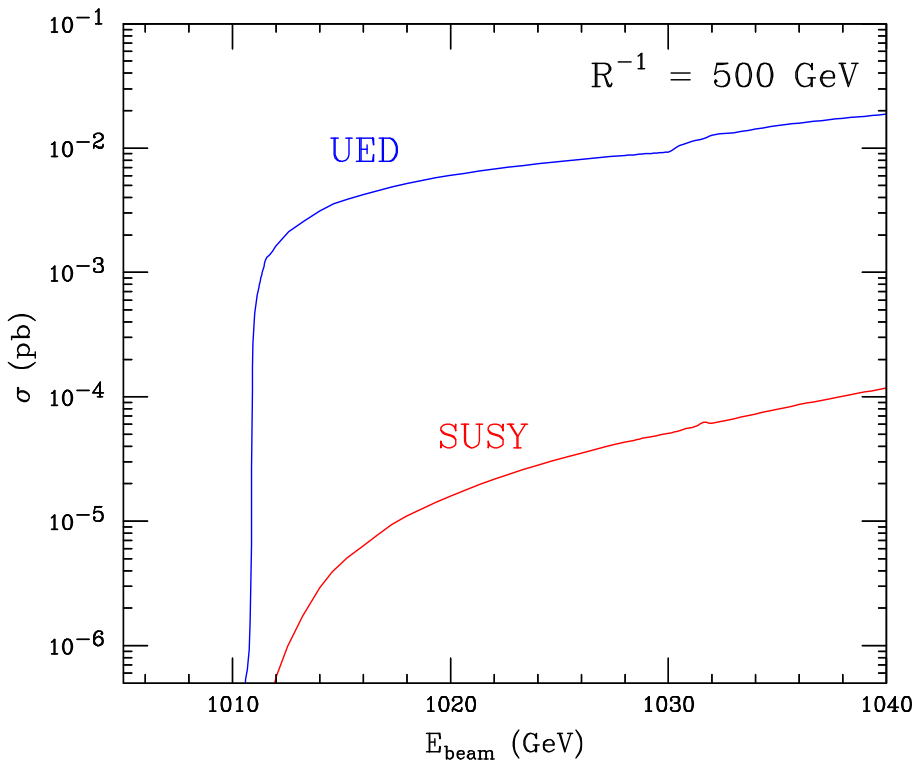




- $E_{max/min} = \frac{1}{2} M_{\mu^*} \left(1 - \frac{M_N^2}{M_{\mu^*}^2}\right) \gamma(1 \pm \beta)$
  - M_{μ^*} : mass of smuon or KK muon
  - M_N : LSP or LKP mass
  - $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ with $\beta = \sqrt{1 - M_{\mu^*}^2/E_{beam}^2}$ (μ^* boost)

Threshold scans (LC)

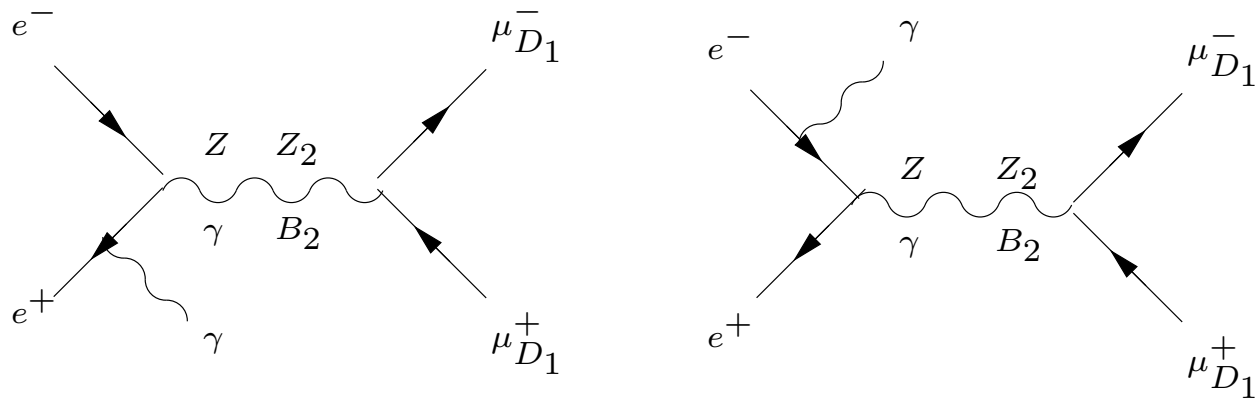
(Battaglia, Datta, De Roeck, Kong, Matchev,hep-ph/0502041)

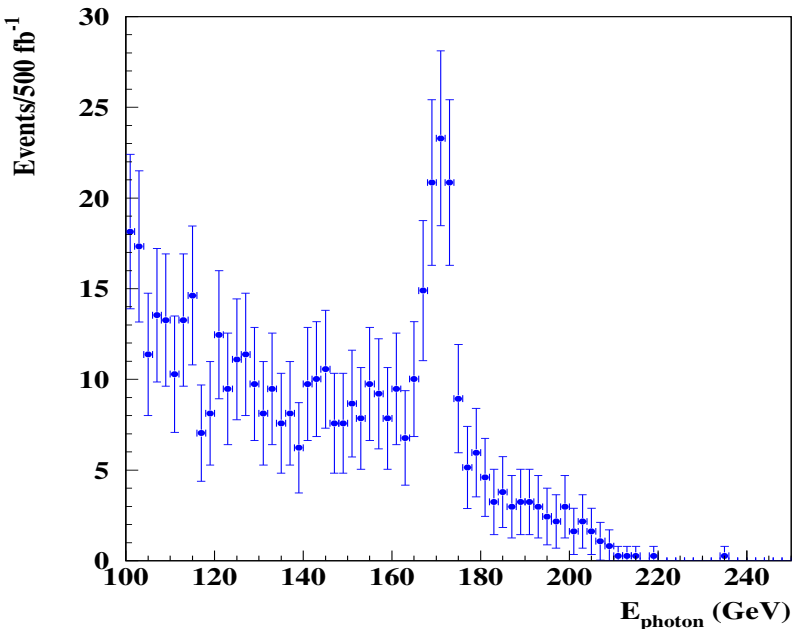


- Mass determination
- Cross section at threshold
  - in UED $\propto \beta$
  - in MSSM $\propto \beta^3 \left(\beta = \sqrt{1 - \frac{M^2}{E_{beam}^2}} \right)$

The Photon Energy Distribution (LC)

(Battaglia, Datta, De Roeck, Kong, Matchev,hep-ph/0502041)

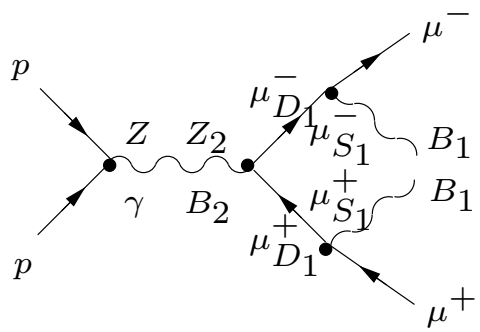


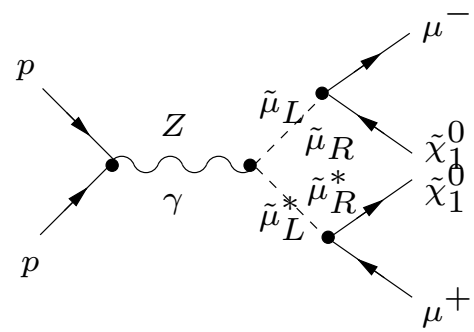


- Smuon production is mediated by γ and Z
- On-shell $Z_2 \longrightarrow \mu_1 \bar{\mu}_1$ is allowed by phase space
- Radiative return due to Z_2 pole at

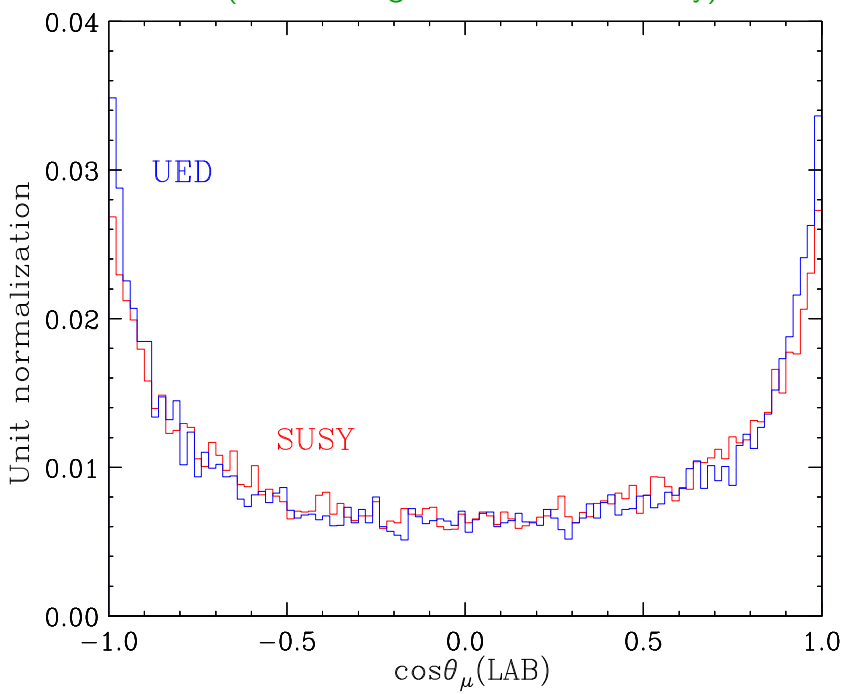
$$E_\gamma = \frac{s - M^2_{Z_2}}{2\sqrt{s}}$$

The Angular Distribution at the LHC



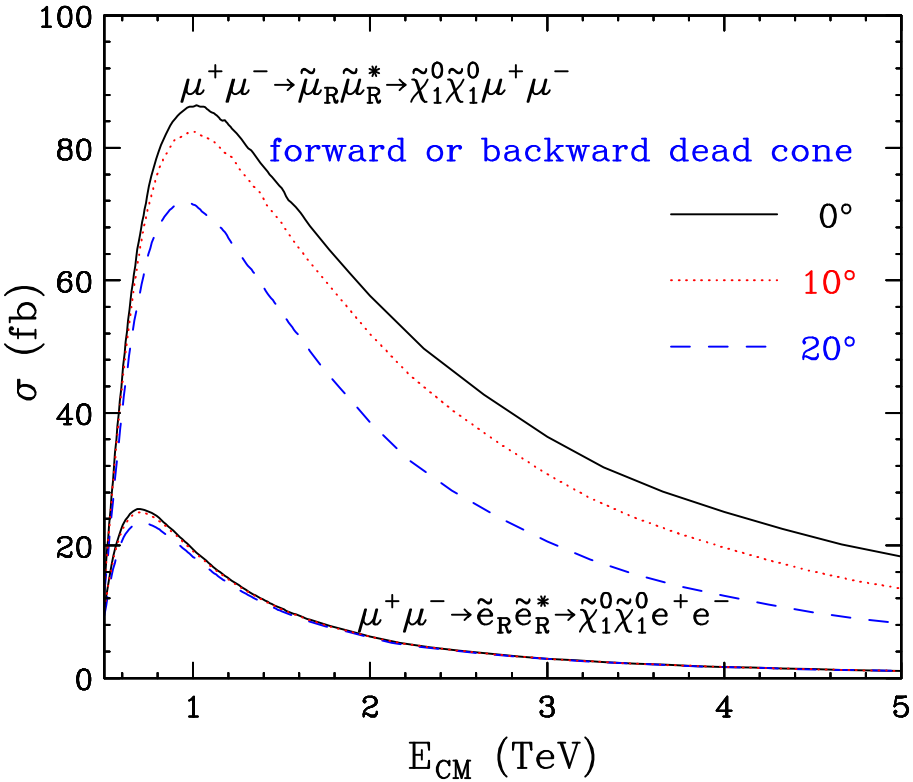


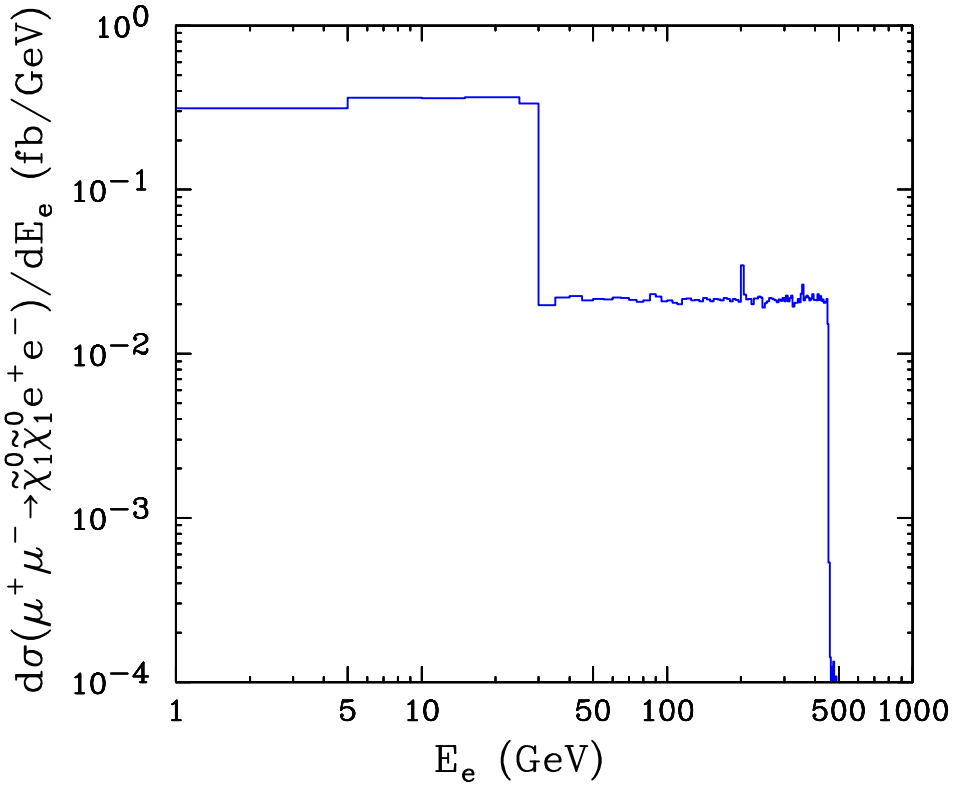
(Datta, Kong, Matchev, Preliminary)



- If we simply do the same trick as in linear collider, it doesn't work
- There is no fixed CM frame

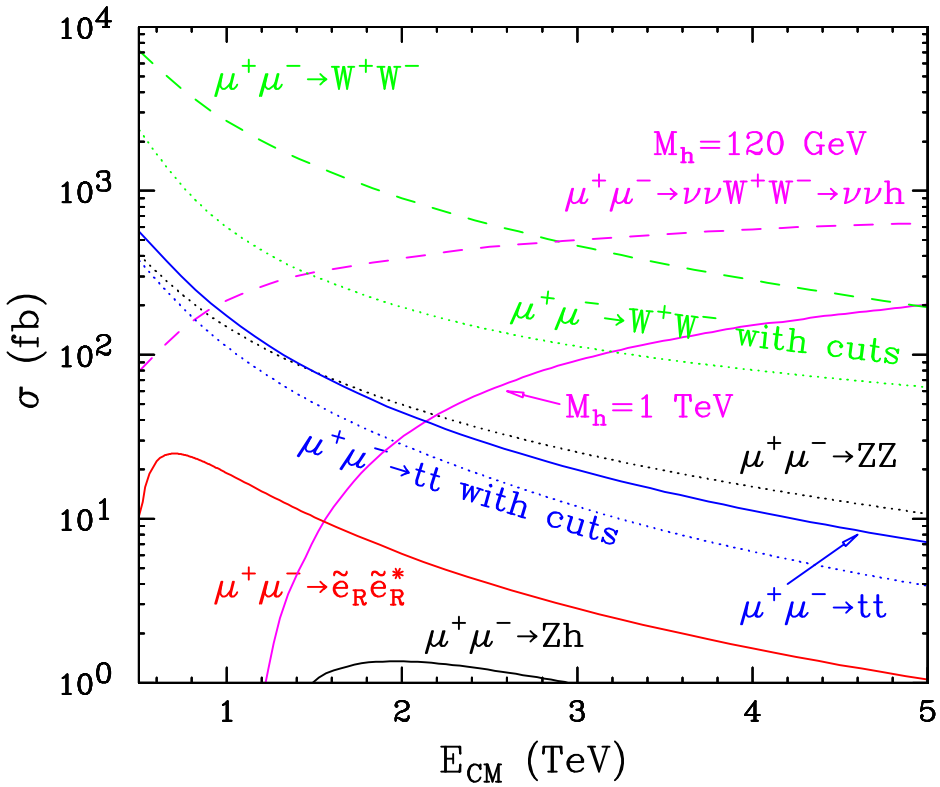
Slepton production at muon collider





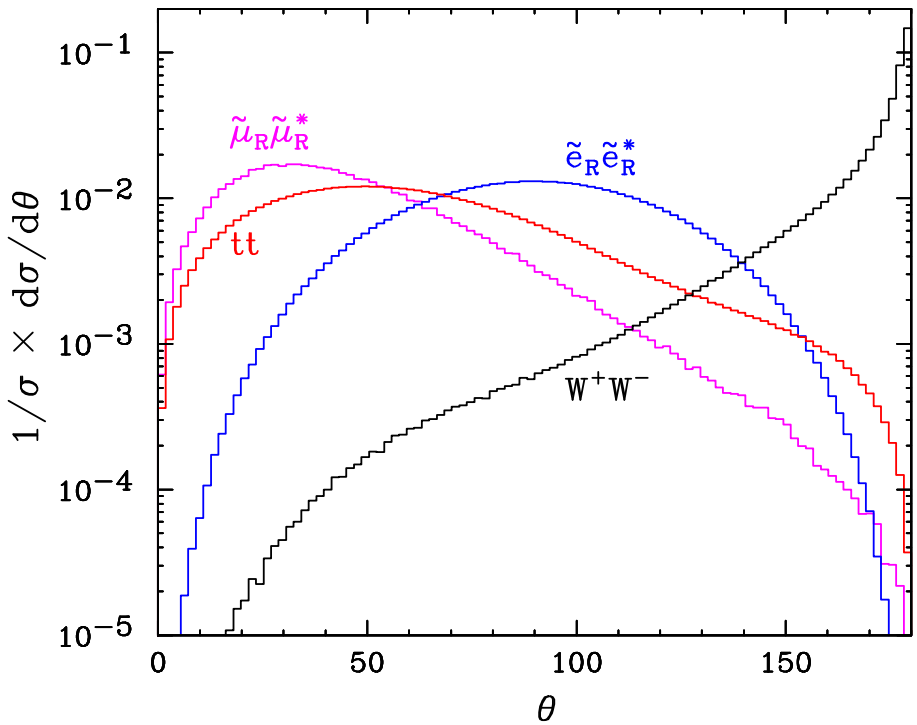
- $\tilde{e}_R\tilde{e}_R^*$: $\sim$ 90% efficiency (10% is lost)
- $\tilde{\mu}_R\tilde{\mu}_R^*$: $\sim$ 50% efficiency (50% is lost)
  smaller efficiency but much larger cross section

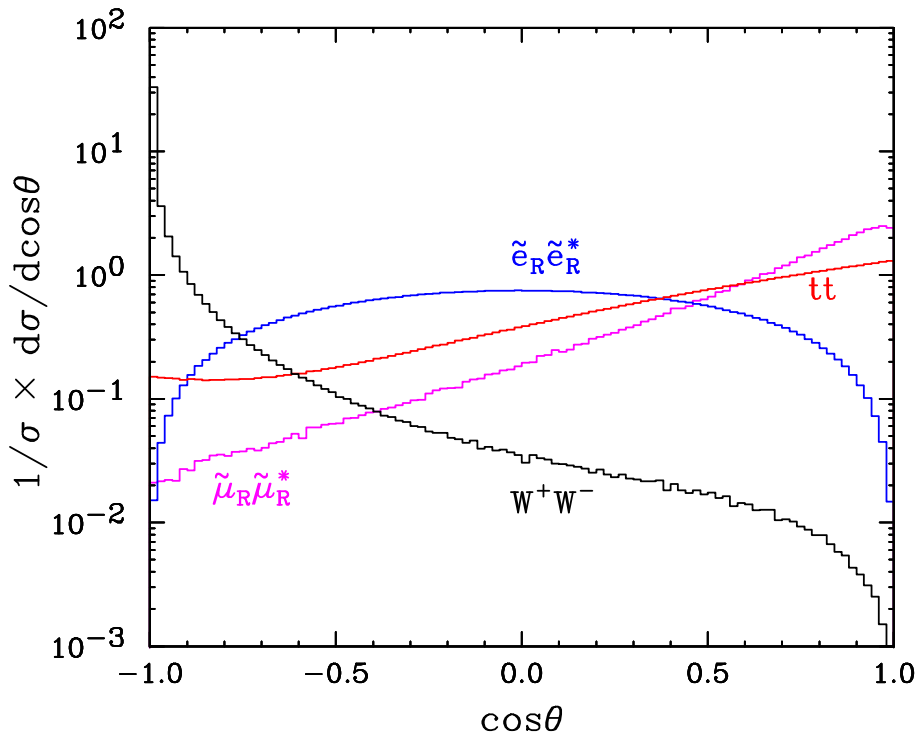
SM production at muon collider



- $\sigma(\ell^+\ell^- \to \nu\bar{\nu}h) \sim \log\left(\frac{s}{m_h^2}\right) \to$ higher energy is better
- Effect of forward/backward lockout
  - $t\bar{t}$: $\sim$ 60% efficiency (40% is lost)
  - W^+W^- : $\sim$ 20% efficiency (80% is lost)

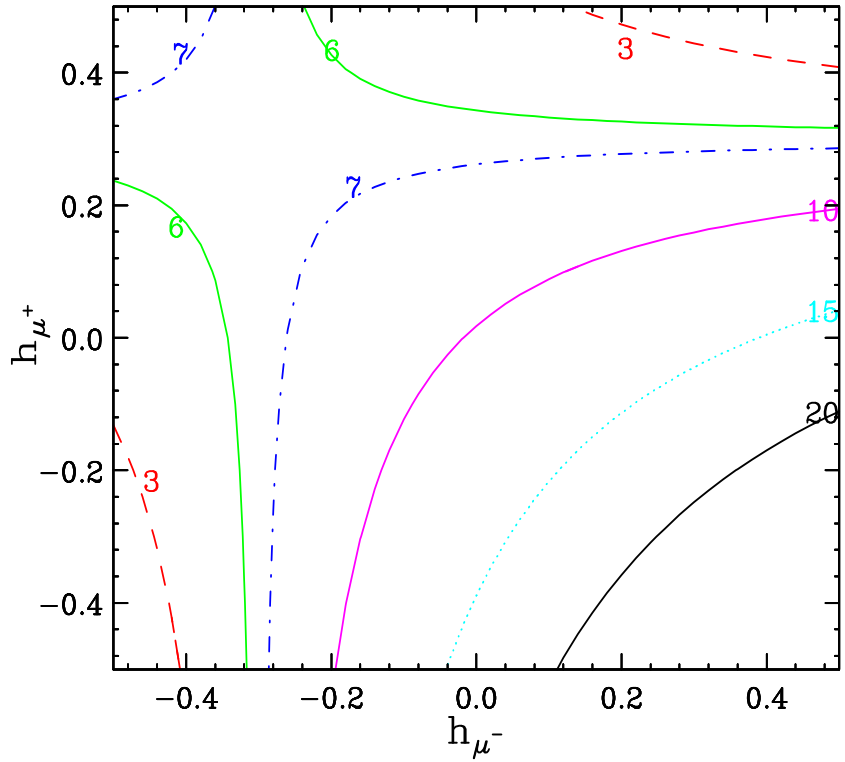
Slepton production at muon collider





- The angular distributions of mother particles
  $\longrightarrow$ angular distributions of daughter particles are correlated
- t-channel $\longrightarrow$ forward or backward
  - Gauge boson pair production, smuon of KK muon production at $\mu^+\mu^-$
- s-channel
  - Fermion pair production $\longrightarrow$ forward or backward
  - Scalar pair production $\longrightarrow$ forward or backward

Polarization



- Polarized beams may enhance signal cross sections and reduce background cross sections, $i.e.$ right handed slepton production and W^+W^- production at ILC, $P(e^-)$=0.8 and $P(e^+)$=-0.6
- Useful to measure particle properties: arXiv:0711.2253, Disentangling CP phases in nearly degenerate resonances: neutralino production via Higgs at a muon collider by Dreiner, Kittel, Pahlen

Summary

- High energy lepton collider is important
  - High energy covers broader range of the particle spectrum
  - lepton collider reveals the true identity of LHC discovery

- Muon collider
  - Option for much higher energy is good
  - Forward/backward lockout effect depends on physics processes
  - Polarization, luminosity, threshold scan, beamstrahlung ...
  - $\gamma\gamma$ or $\mu^-\mu^-$ colliders?

Beamstrahlung at Muon Collider?

- Beamstrahlung parameter:

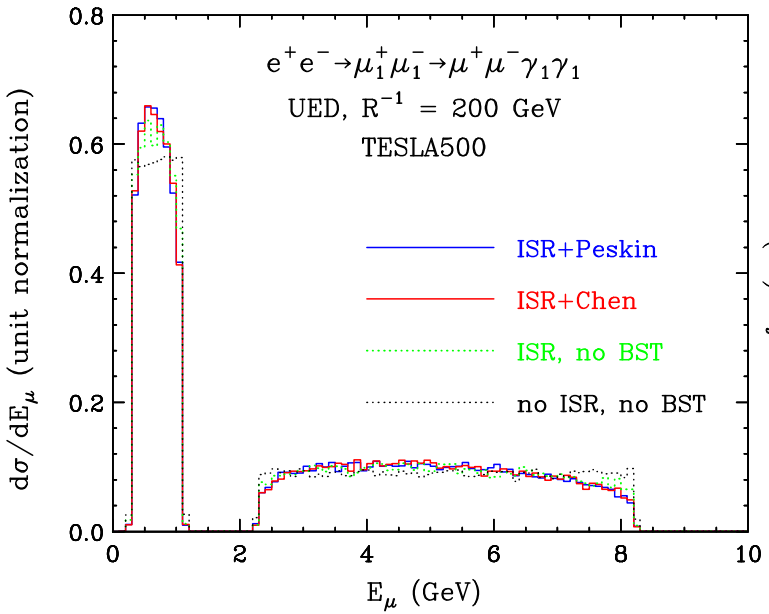
$$\Upsilon \approx \frac{5}{6}\frac{\alpha E_{beam} N}{m^3 \sigma_z (\sigma_x + \sigma_y)}$$

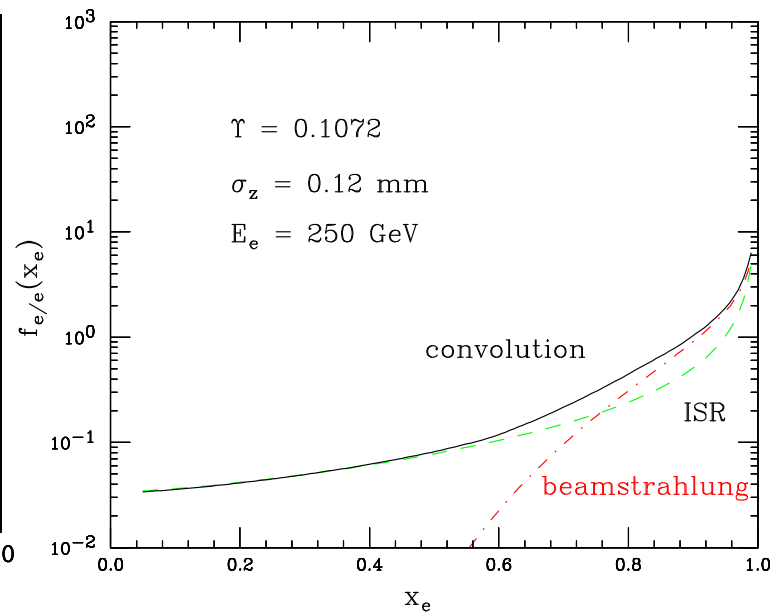
	CLIC 3 TeV	Muon Collider 3 TeV
Number of particles/bunch (N)	4×10^9	6.25×10^9
length (σ_z)	35 μm	0.5 cm
beam sizes (σ_x/σ_y)	60/0.7 nm	?/?
mass (m)	0.5 MeV	100 MeV

- Most LCs: $\Upsilon \ll 1$ or $\Upsilon < 1$
  - SLC, $\Upsilon \sim 0.004$ (Chen, Phys. Rev. D46,1186(1992))
  - NLC500: $\Upsilon = 0.104$, NLC1000, $\Upsilon = 0.299$ (Peskin LCC-0010)
  - CLIC3000: $\Upsilon = 5$, CLIC5000: $\Upsilon = 10$ (CLIC Physics Working Group Report)
- Naively $\Upsilon \ll 1$ expected at Muon collider
  (Muons are more complicated due to higgs coupling)
- Good: less (two photons) background/less energy loss$\longrightarrow$better measurement
- Bad: no beamstrahlung or ISR photon

E_μ distribution at colliders with $\Upsilon < 1$

Datta, Kong, Matchev (preliminary)

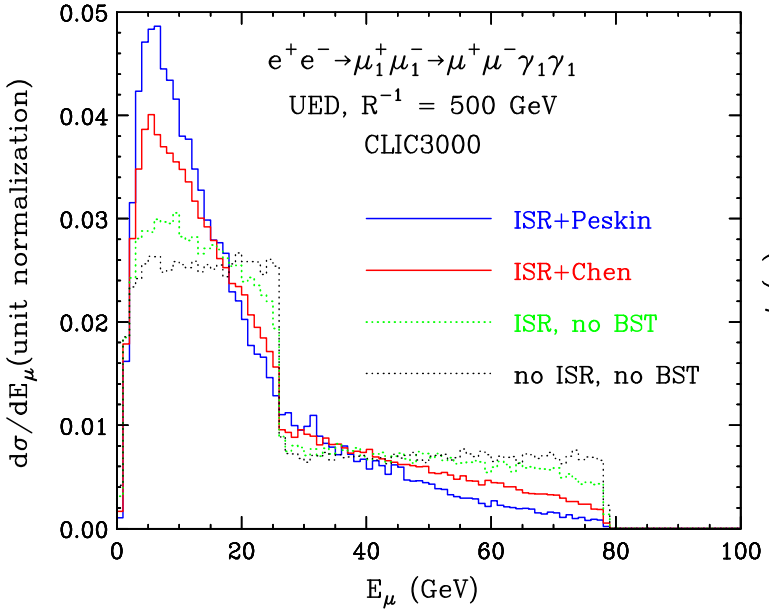


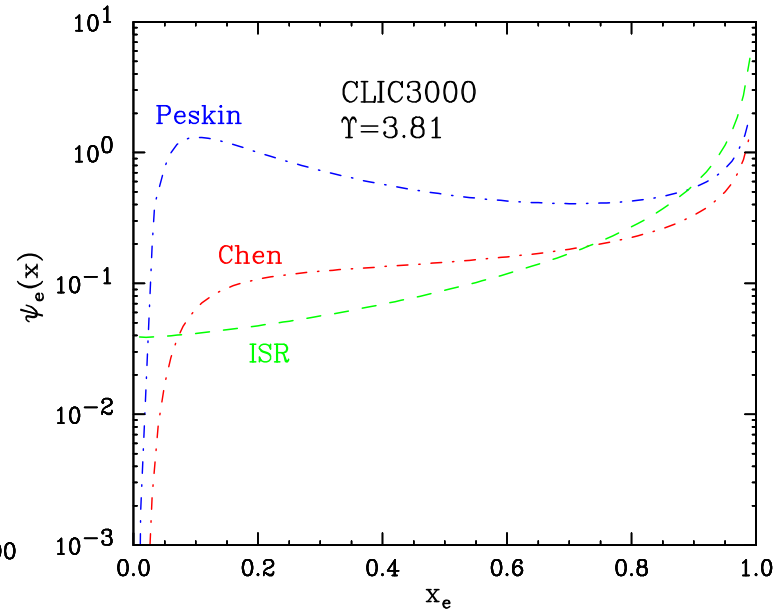


- End points are determined by the kinematics: sensitive to masses and energy
  - $E_{max/min} = \frac{1}{2} M_{\tilde{\mu}} \left(1 - \frac{M^2_{\tilde{\chi}^0_1}}{M^2_{\tilde{\mu}}} \right) \gamma(1 \pm \beta)$
- Exactly flat distribution is expected if no energy loss
- ISR and beamstrahlung distribute the energies of incoming electrons
  $\longrightarrow$ smear the distribution $\longrightarrow$ endpoints still OK
- ISR is more important at low x since beamstrahlung drops fast

E_μ distribution at colliders with $\Upsilon > 1$

Datta, Kong, Matchev (preliminary)



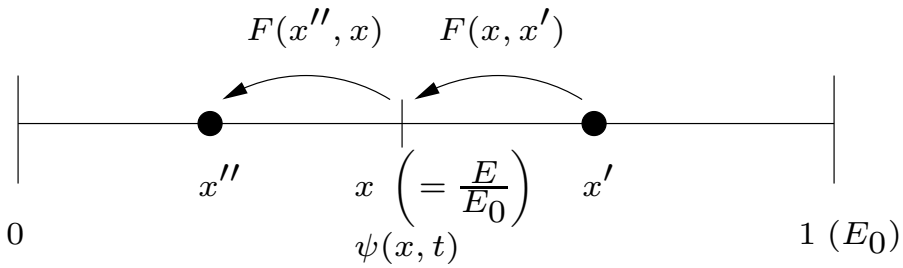


- Can't see nice flat distribution $\longrightarrow$ distorted a lot
- Consistent YC deviates more since it has a peak at low x
- Beamstrahlung is more important at low x
- Flat tail or peak at low x (soft photons) affect physical distribution
- Is there a peak ? Which is the right answer ?

Beamstrahlung

(Sokolov and Ternov 1986)

- Rate equation



  - $\frac{\partial \psi}{\partial t} = -\underbrace{\int_0^x dx'' F(x'', x)}_{\equiv \nu(x)} \psi(x,t) + \int_x^1 dx' F(x, x') \psi(x', t)$
  - $\frac{\partial \psi}{\partial t} = \nu(x)\psi(x,t) + \int_x^1 dx' F(x, x') \psi(x', t)$

- $F(x, x') = \frac{\nu_{cl} \kappa}{x x'} f(\xi, \eta)$

  $f(\xi, \eta) = \frac{3}{5\pi} \frac{1}{1+\xi\eta} \left[\int_\eta^\infty du K_{5/3}(u) + \frac{\xi^2 \eta^2}{1+\xi\eta} K_{2/3}(\eta) \right]$

  - $\xi = 3x'\Upsilon$, $\eta = \kappa \left(\frac{1}{x} - \frac{1}{x'}\right)$, $\kappa = \frac{2}{3\Upsilon}$
  - $\Upsilon = \frac{5}{6} \frac{r_e^2 \gamma_0 N}{\alpha \sigma_z (\sigma_x + \sigma_y)}$
  - $\nu_{cl} = \frac{5}{2\sqrt{3}} \frac{\alpha^2}{r_e \gamma_0} \Upsilon =$ constant

Yokoya-Chen approximation

Yokoya and Chen, IEEE 1989, SLAC-PUB-4935 and Chen, Phys. Rev. D46,1186(1992)

- Can't solve rate equation analytically $\longrightarrow$ Need approximations to solve
- Most LCs have $\Upsilon \ll 1$ or $\Upsilon < 1$ $\left(\Upsilon \sim \frac{\gamma_0 N}{\sigma_z(\sigma_x+\sigma_y)}\right)$
  - SLC, $\Upsilon \sim 0.004$ (Chen, Phys. Rev. D46,1186(1992))
  - NLC500: $\Upsilon = 0.104$, NLC1000: $\Upsilon = 0.299$ (Peskin LCC-0010)
  - CLIC3000: $\Upsilon = 5$, CLIC5000: $\Upsilon = 10$
- Two assumptions (valid for $\Upsilon \ll 1$)
  - $f(\xi,\eta) = \frac{3}{5\pi}\frac{1}{1+\xi\eta}\left[\int_\eta^\infty duK_{5/3}(u) + \frac{\xi^2\eta^2}{1+\xi\eta}K_{2/3}(\eta)\right]$
    $\approx \frac{1}{\Gamma(1/3)}\eta^{-2/3}e^{-\eta}$
  - $\nu(x) = \int_0^x dx'' F(x,x'') \approx \nu_{cl} =$ constant
- Analytic solution is possible by Laplace transformation
  - $\psi(x) = \frac{1}{N_{cl}}\left[(1-e^{-N_{cl}})\delta(1-x) + \frac{e^{-\eta x}}{1-x}\bar{h}(x)\right]$
- Good agreement with simulation data (ABEL) up to $\Upsilon \sim 0.44$
  (Chen, Phys. Rev. D46,1186(1992))

Consistent YC approximation

Peskin LCC-0010, SLAC-TN-04-032

- Incompatibility of two assumptions
  - $\tilde{f}(\eta) = \frac{1}{\Gamma(1/3)}\eta^{-2/3}e^{-\eta} \longleftrightarrow \nu(x) = \text{constant}$
- Normalization condition: $\int_0^1 dx\psi(x,t) = 1$
- Need better $\tilde{f}$ - Peskin : $\tilde{f}(\eta) = \frac{x'}{x}\frac{1}{\Gamma(1/3)}\eta^{-2/3}e^{-\eta}$
- Preserves probability sum rule
- Analytic solution
  - $\psi(x) = e^{-N}\left(\delta(x-1) + \frac{e^{-\eta x}}{x(1-x)}h(N\eta_x^{1/3})\right)$
- Numerically not very different from previous solution ($\Upsilon < 1$)
- Good agreement with Guinea Pig simulation for $\Upsilon = 0.104$ (NLC500) and $\Upsilon = 0.299$ (NCL1000)

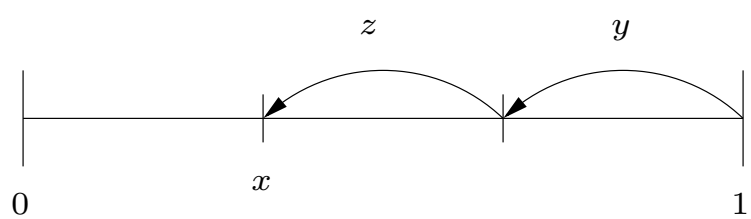
Bremstrahlung and convolution

- Bremstrahlung (ISR)

  Kuraev/Fadin (1985), Jadach/Skrzypek (1991), Jadach/Ward (1990)

  - $f_{ISR}(x) = \frac{1}{2}\beta(1 - x^{\frac{\beta}{2}} - 1)(1 + \frac{3}{8}\beta) - \frac{1}{4}\beta(1 + x)$
  - $\beta = \frac{2\alpha}{\pi}\left(\log\frac{Q^2}{m_e^2} - 1\right)$

- Convolution

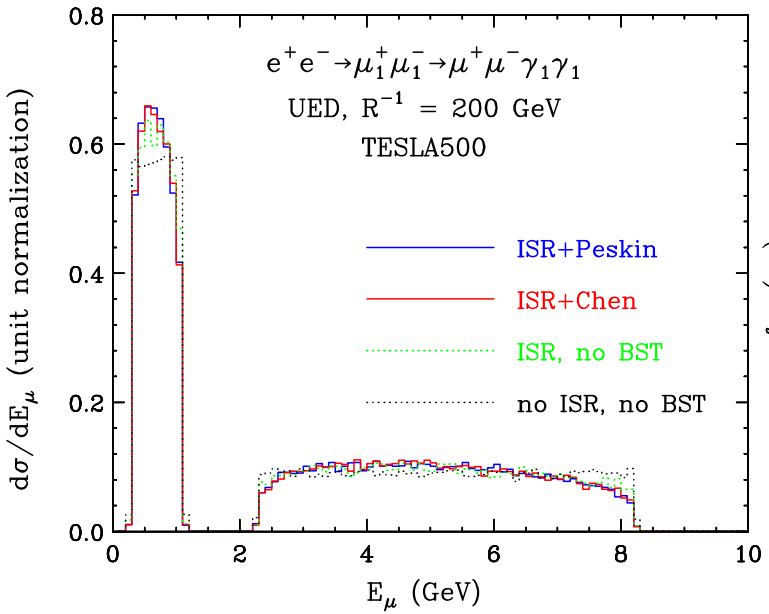


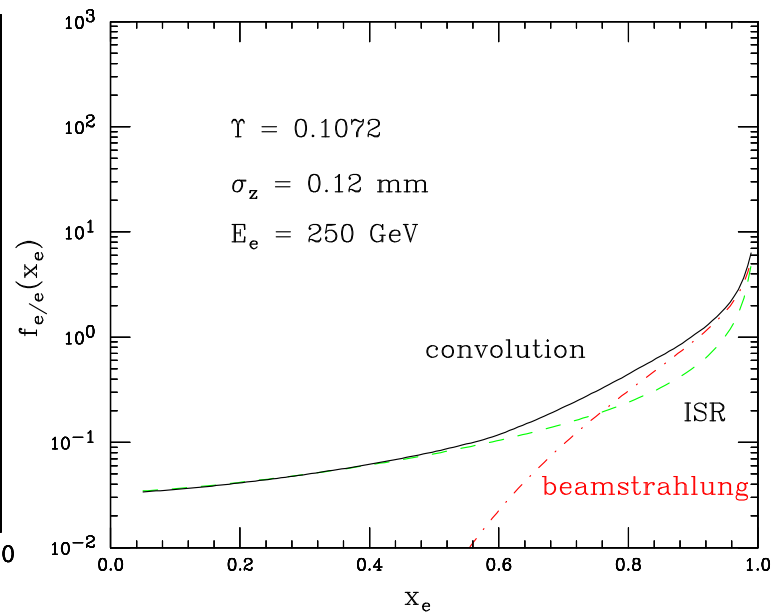
  - $f_1 * f_2 = \int_0^1 dydz f_1(y) f_2(z) \delta(x - yz)$
  - $f_{ISR} * f_{beam} = f_{beam} * f_{ISR}$

    $= \int_x^1 \frac{dz}{z} f_{ISR}\left(\frac{x}{z}\right) f_{beam}(x)$

E_μ distribution at colliders with $\Upsilon < 1$

Datta, Kong, Matchev (preliminary)

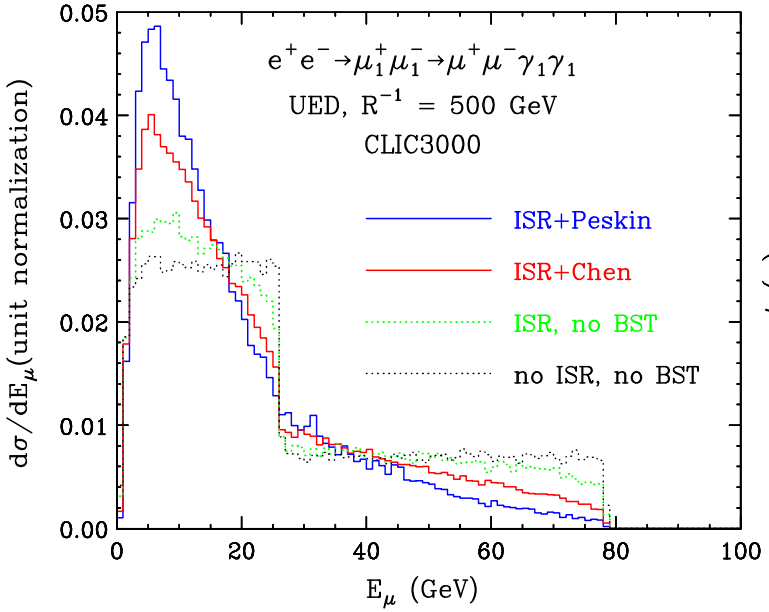


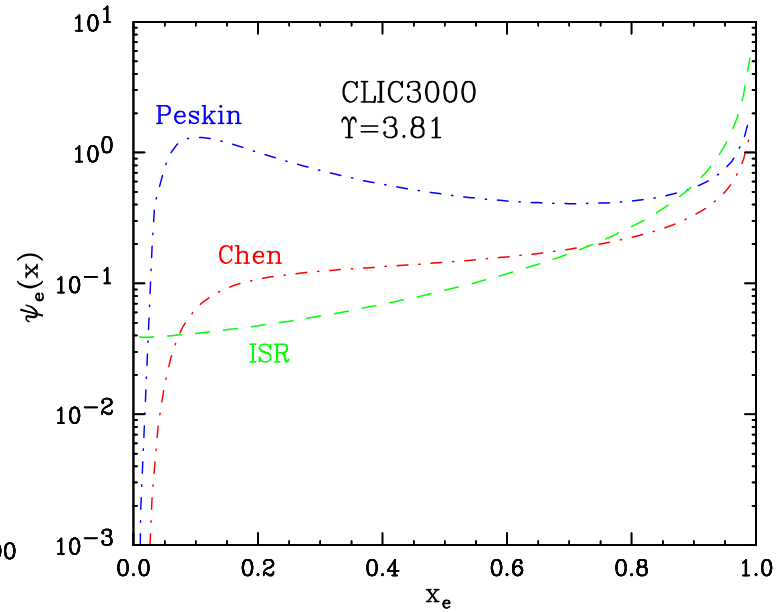


- End points are determined by the kinematics: sensitive to masses and energy
  - $E_{max/min} = \frac{1}{2} M_{\tilde{\mu}} \left(1 - \frac{M^2_{\tilde{\chi}^0_1}}{M^2_{\tilde{\mu}}} \right) \gamma(1 \pm \beta)$
- Exactly flat distribution is expected if no energy loss
- ISR and beamstrahlung distribute the energies of incoming electrons
  $\longrightarrow$ smear the distribution $\longrightarrow$ endpoints OK
- ISR is more important at low x since beamstrahlung drops fast

E_μ distribution at colliders with $\Upsilon > 1$

Datta, Kong, Matchev (preliminary)



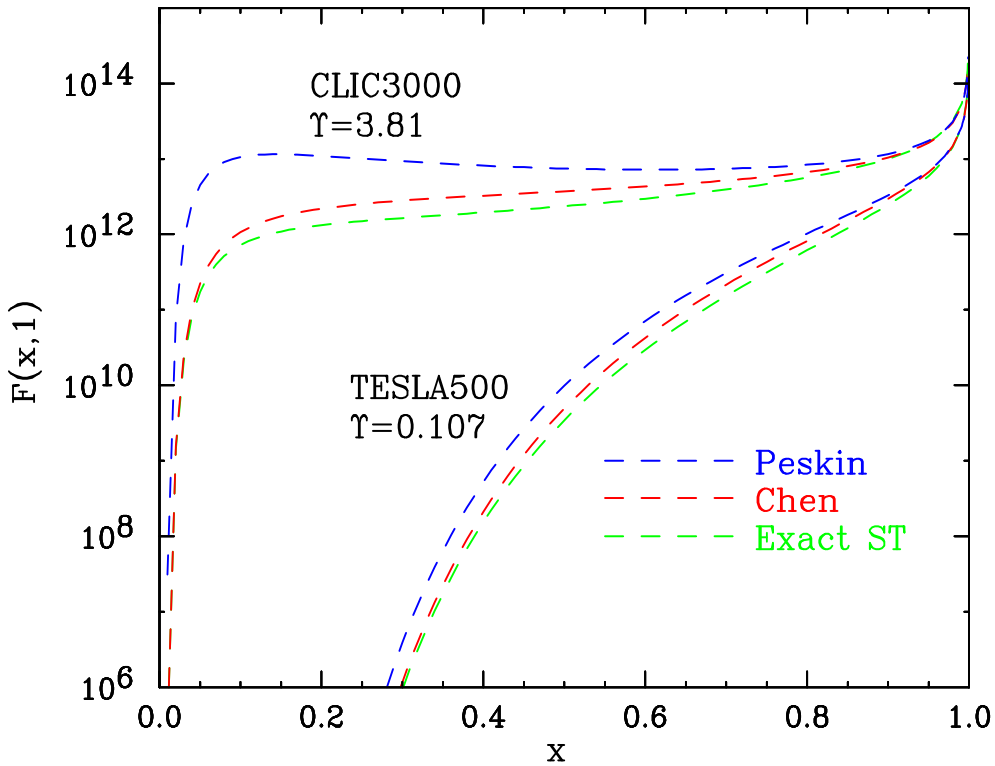


- Can't see nice flat distribution $\longrightarrow$ distorted a lot
- Consistent YC deviates more since it has a peak at low x
- Beamstrahlung is more important at low x
- Flat tail or peak at low x (soft photons) affect physical distribution
- Is there a peak ? Which is the right answer ?

Applicability of approximations for $\Upsilon > 1$

- $\tilde{f}(\eta) = \frac{x'}{x}\frac{1}{\Gamma(1/3)}\eta^{-2/3}e^{-\eta}$ is only valid for $\Upsilon \ll 1$
- Original approximation is better

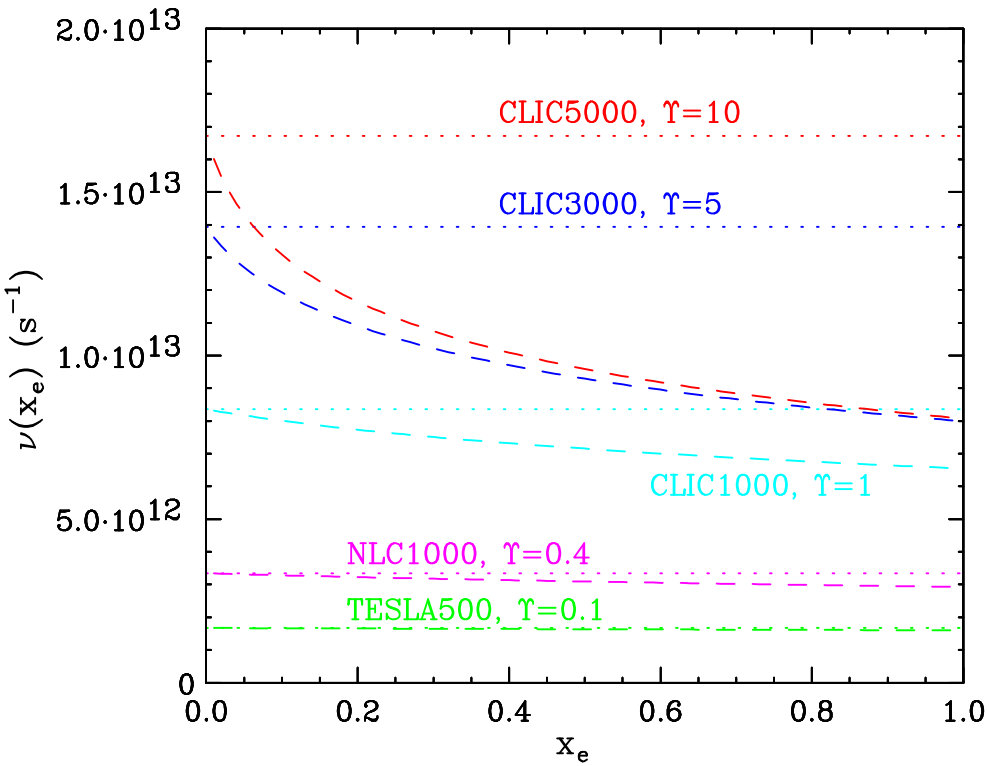
Datta, Kong, Matchev (preliminary)



Applicability of approximations for $\Upsilon > 1$

- $\nu(x) = \int_0^x F(x'', x) dx'' \neq$ constant

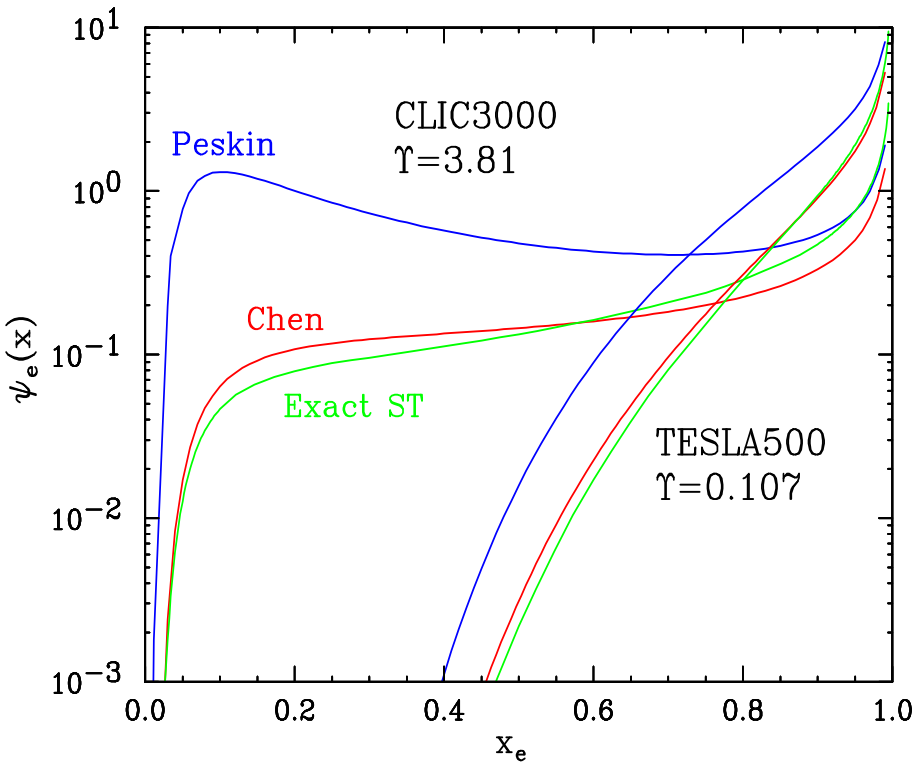
Datta, Kong, Matchev (preliminary)



- For large Υ (CLIC), we can trust neither approximations
- Need new formula/better approximation (couldn't find)

Numerical solution

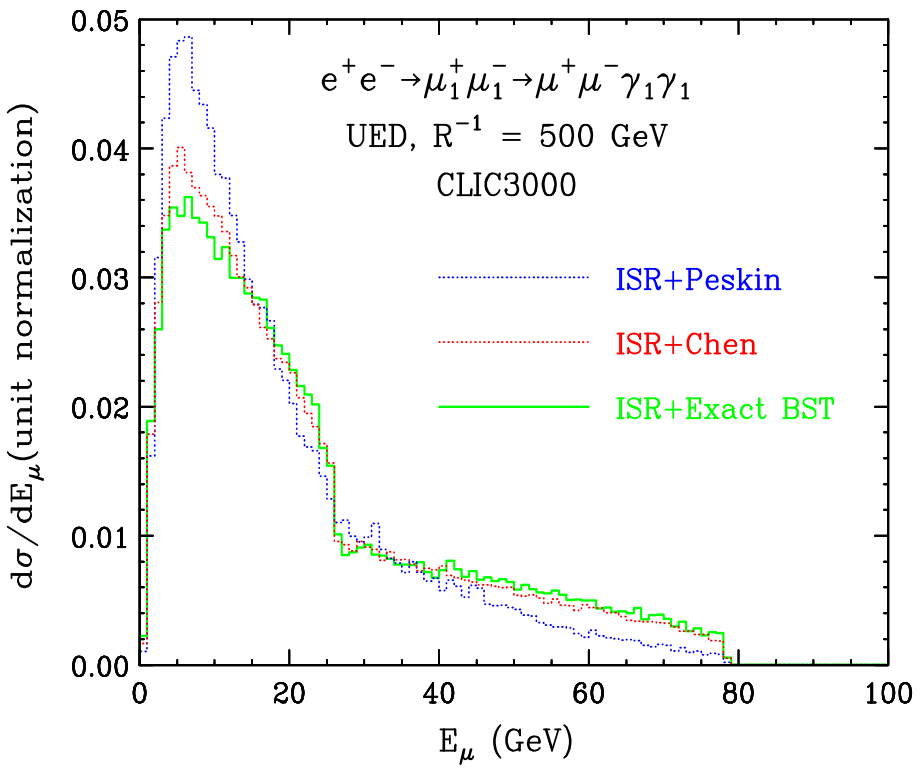
Datta, Kong, Matchev (preliminary)



- Solve the rate equation directly with Sokolov-Ternov solution
  - Good : valid for arbitrary Υ, normalized
  - Bad : no analytic expression
    $\longrightarrow$ need to modify the event generator code
- Chen is closer for identical parametrization
- Roughly agrees with old simulation data.
  We will compare to new data from Schulte

E_μ distribution at colliders with $\Upsilon > 1$

Datta, Kong, Matchev (preliminary)



- Still distorted
- We know the right answer