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Innovation in Research

PIC/REMEX Lattice Update

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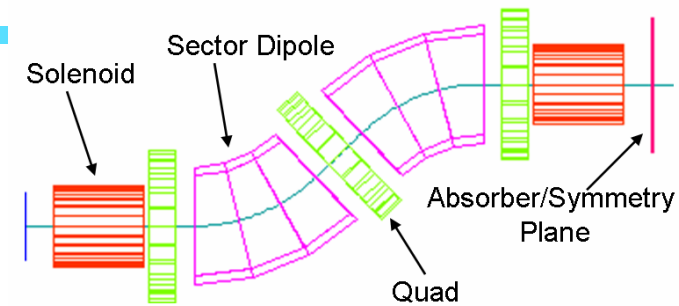
MCTF Weekly Meeting
26 July 2007

New Basic Lattice … Why?



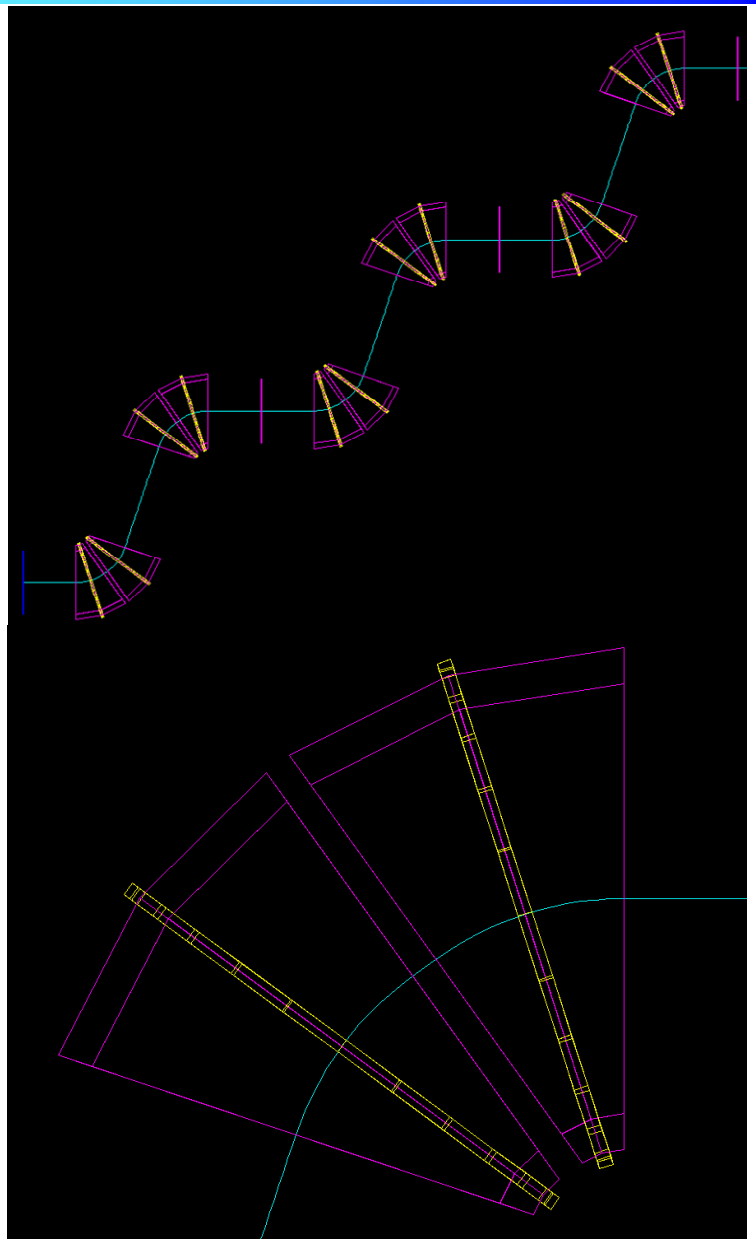
- Solenoids presented difficulties
  - End Fields
    - Realistic models has large longitudinal extent.
    - Practical models for simulations was not physically realizable.
  - Strong possibility of x-y coupling without careful control.
  - Based on β-function concept and analysis
    - Lens focusing achieves small spot size (large divergence) by reducing the local β-function .
    - Even a stable (perfect) ½-integer lattice does not have a defined β-function
  - Large aberrations caused significant phase space distortion.
- New lattice concept is simpler, and more directly based on the basic equations.

$$x'' + (\kappa^2 - n)x = \kappa(s)\frac{\Delta P}{P}$$

$$y'' + ny = 0$$

New PIC Lattice Design

- Only uses dipoles and quadrupole magnets with no "fringe" fields.
- Each bend (½-cell) consists of 2 sector dipoles and 2 thin quads that act in unison.
  - All bends have the same angle (dipole field) only the bend direction changes.
  - All quads have the same unperturbed field.
  - Quads are thin to prevent physical interference at the short dipole side and sagitta effects.
  - Quads encircle the dipoles (see later figures).
- 2 independent quantities:
  - Bend angle – field automatically adjusted by G4Beamline
  - Quadrupole field gradient
- Y tune is only affected by quadrupole gradient
- X tune is affected by both quad gradients and bend angle.
- Fringe field effects will change parameter settings, but not cause any coupling between the x and y motion. Thus, the x and y tune will be independent.



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Calculate Tune

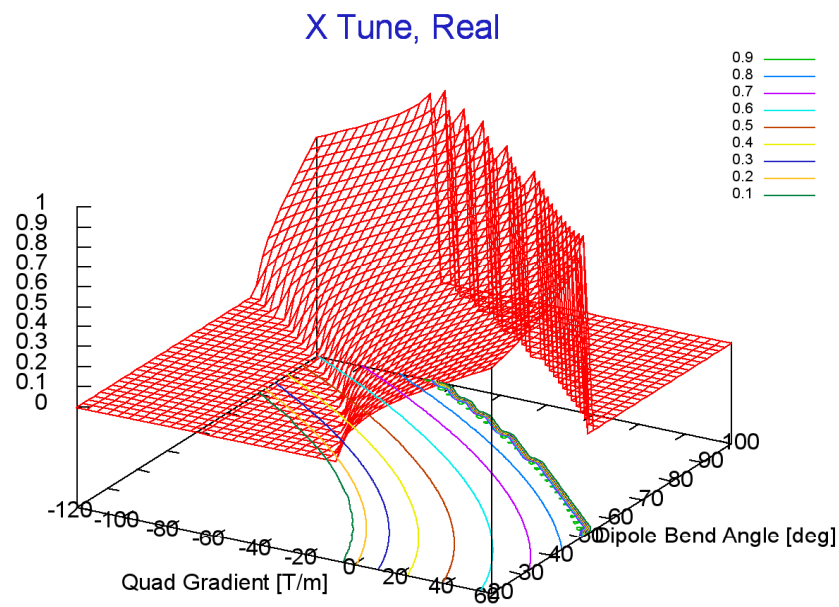
- Track displaced particles and calculate transport (Jacobian) matrix.
- 2 sets of reciprocal complex eigenvalues.
- 2 sets of complex conjugate eigenvectors.
- Stable eigenvalues fall on a unit circle in the complex plane ($Q_i = 0$).
- Q_r is defined modulo 1.
- Chosen eigenvalue is consistent with the formalisms of Courant-Snyder and the Generalized 4D Twiss formalism of Lebedev & Bogacz.
- Positive imaginary tune means amplitude growth.
- Minor numerical coupling causes full 4D values to be slightly different from block diagonal values.

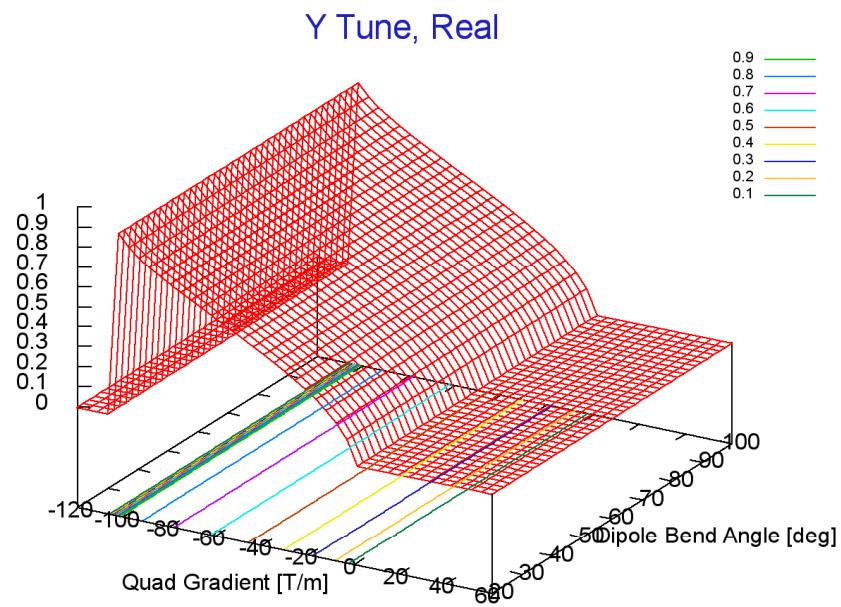
$$M = \begin{pmatrix} \frac{\Delta x_f}{\Delta x} & \frac{\Delta x_f}{\Delta p_x} & \frac{\Delta x_f}{\Delta y} & \frac{\Delta x_f}{\Delta p_y} \\ \frac{\Delta p_{x,f}}{\Delta x} & \frac{\Delta p_{x,f}}{\Delta p_x} & \frac{\Delta p_{x,f}}{\Delta y} & \frac{\Delta p_{x,f}}{\Delta p_y} \\ \frac{\Delta y_f}{\Delta x} & \frac{\Delta y_f}{\Delta p_x} & \frac{\Delta y_f}{\Delta y} & \frac{\Delta y_f}{\Delta p_y} \\ \frac{\Delta p_{y,f}}{\Delta x} & \frac{\Delta p_{y,f}}{\Delta p_x} & \frac{\Delta p_{y,f}}{\Delta y} & \frac{\Delta p_{y,f}}{\Delta p_y} \end{pmatrix}$$

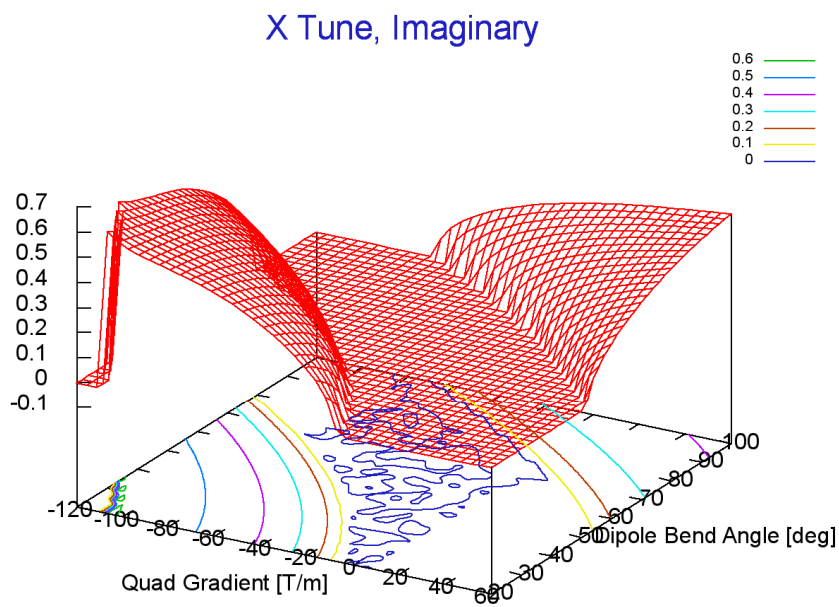
$$\lambda = e^{-i2\pi Q} \Rightarrow$$

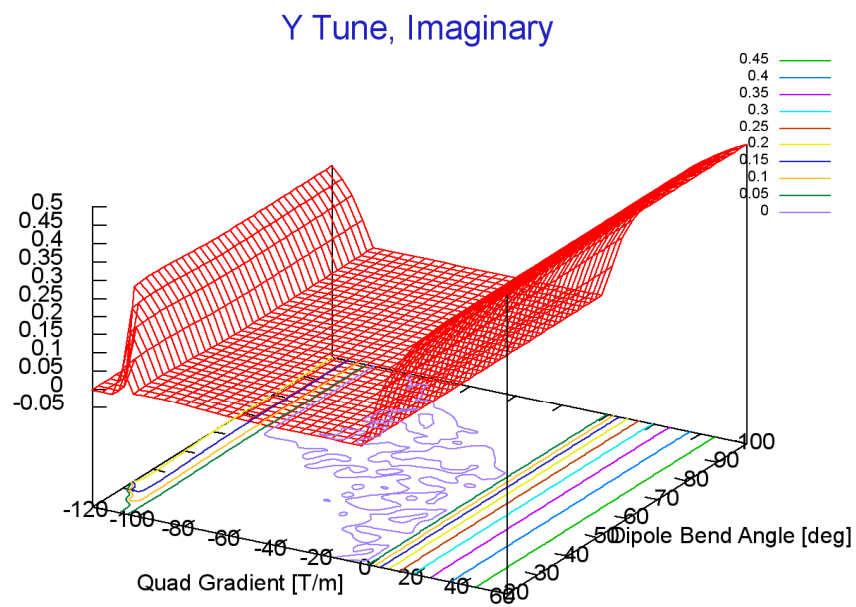
$$= e^{2\pi Q_i} e^{-i2\pi Q_r}$$

Full 4D Tune Profile

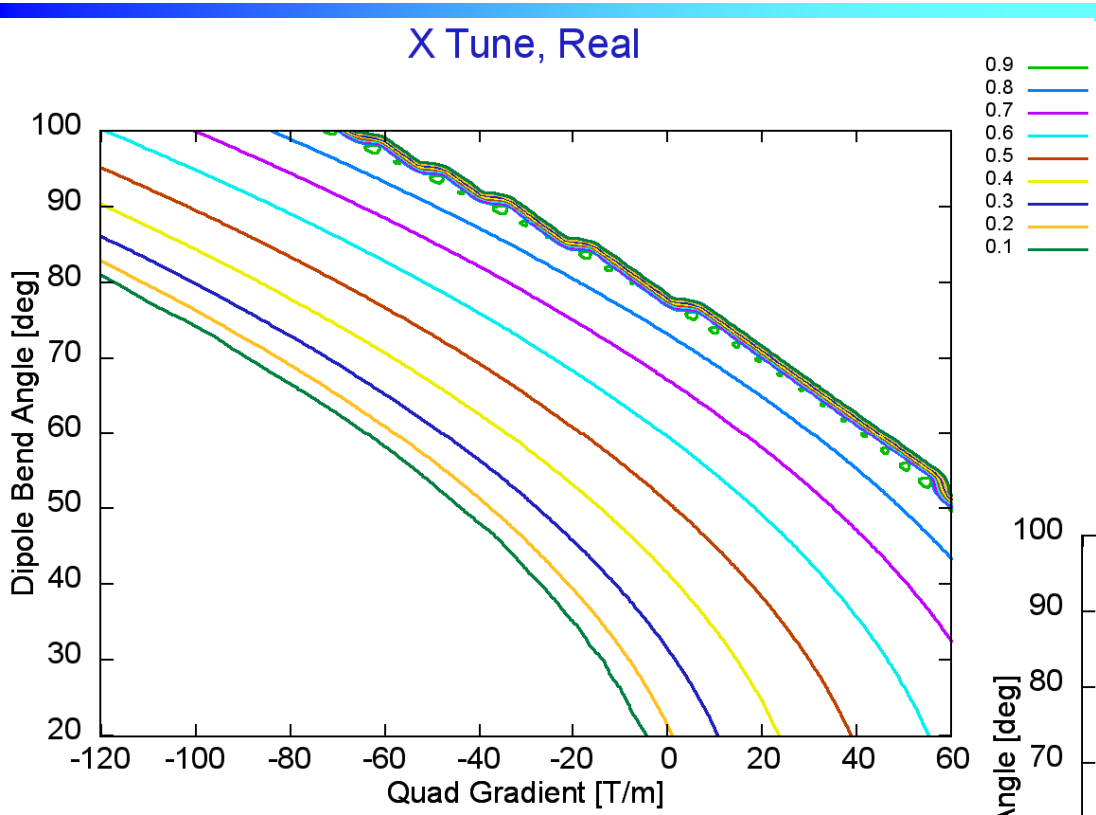








Real Tunes



½-Integer Tune (G4Beamline):

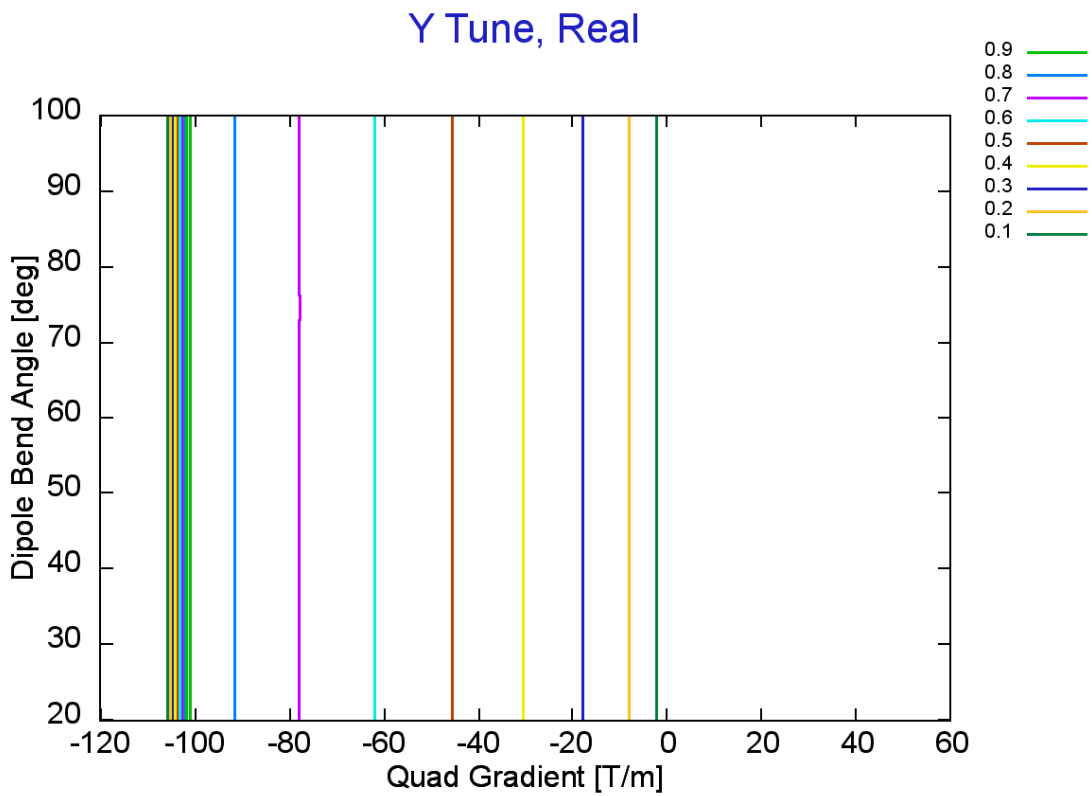
G_Q = -45.8100 T/m

θ = 71.4887°

½-Integer Tune (Thick Lens Matrix):

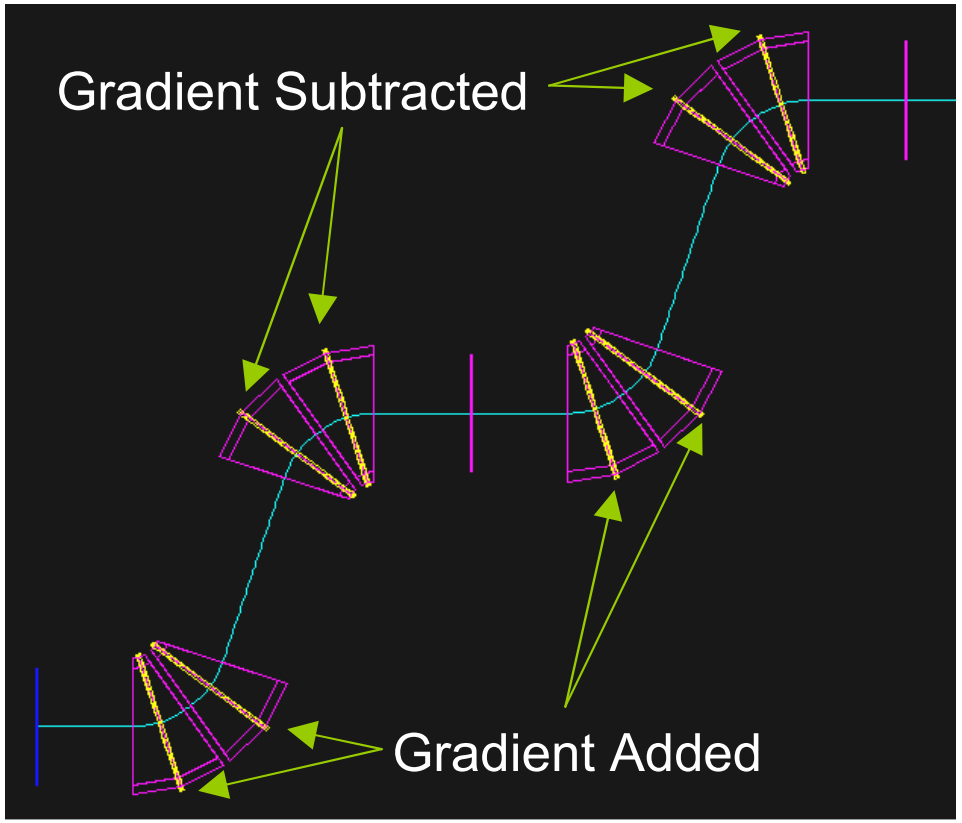
G_Q = -45.8068 T/m

θ = 71.4818°

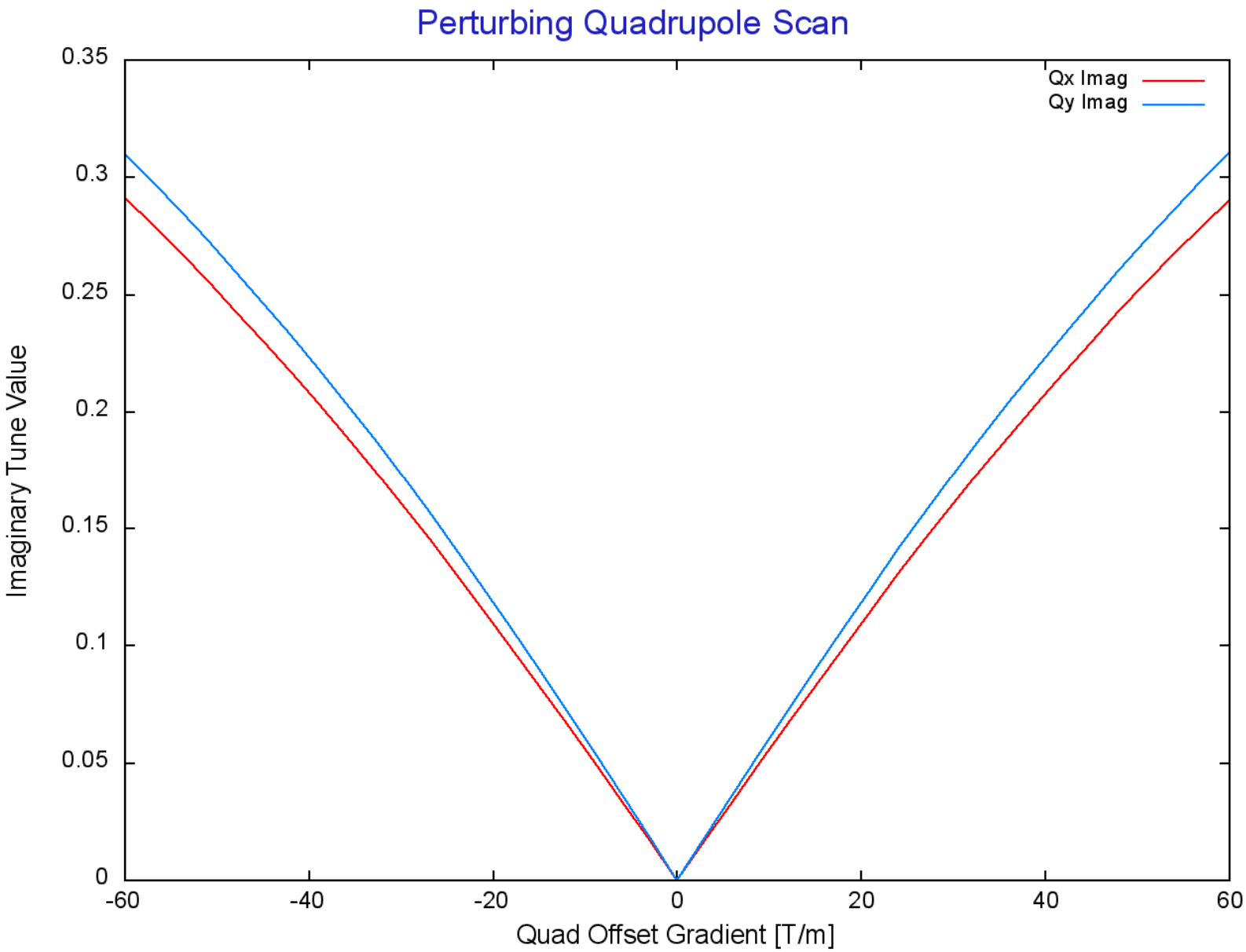


Add Perturbation

- Field gradient perturbation added or subtracted to quads in place (no new quads added).
- Dipole “blocks” act together.
- Alternate blocks varied in opposite fashion.
- Perturbation can be positive or negative.
- Initial perturbation values are *significantly* larger than those seen later.
- No elements were added.



Imaginary Tune Change

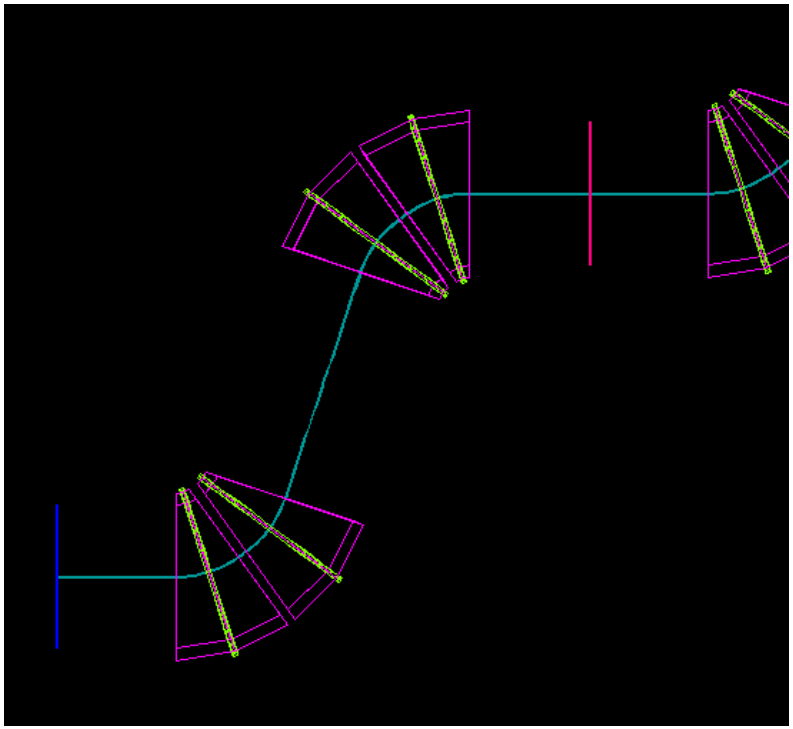


- Nominal G_Q = -45.81 T/m
- Real Tune = 0.50 in all cases
- Large range used to show linear trend of growth rate with perturbation
- Consistent with parametric resonance
- $Q_i \approx 0.16$ corresponds to an e-folding.

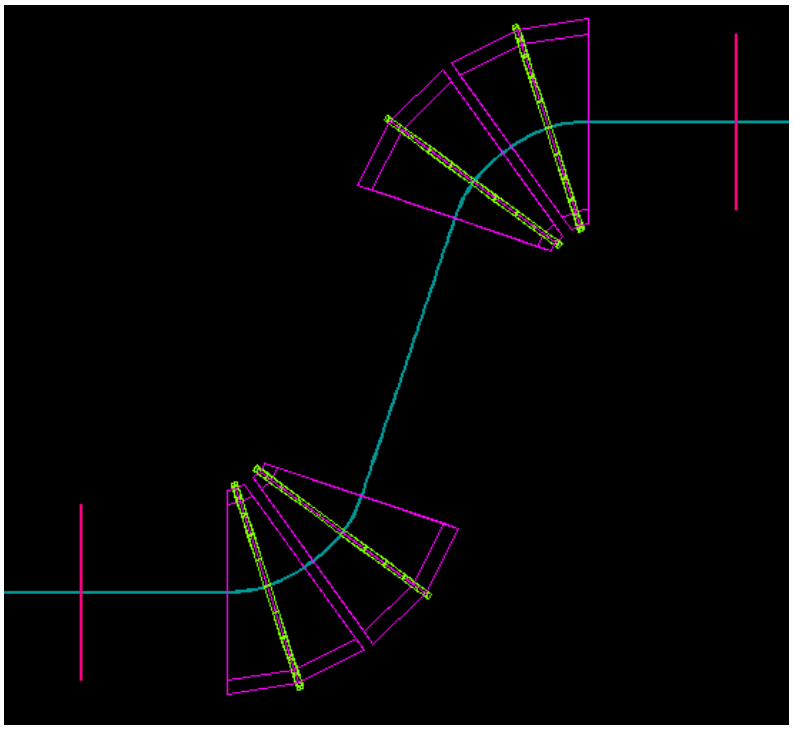
20 Cell (ΔG_Q=0 T/m)

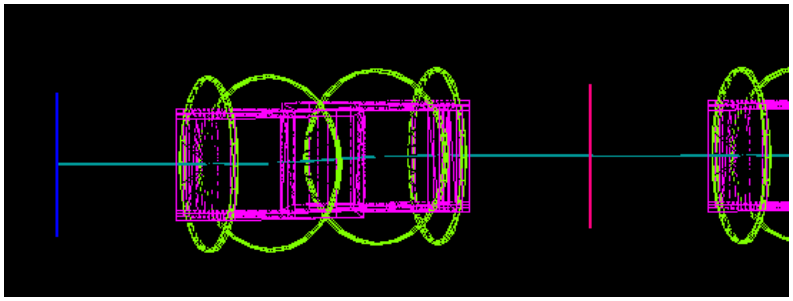
Cell 1

Cell 20

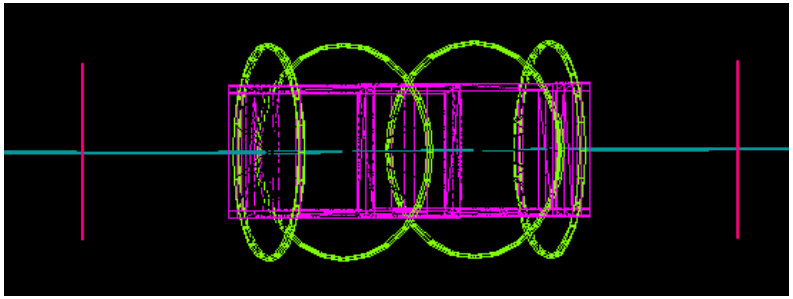


X





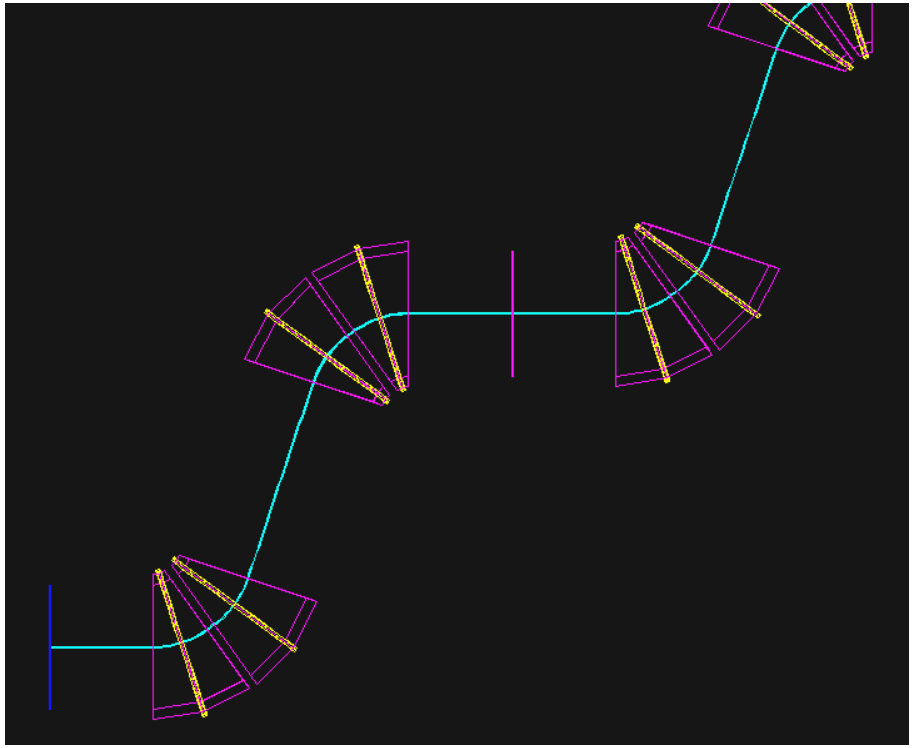
Y



20 Cell (ΔG_Q=5 T/m)

Cell 1

Cell 19-20



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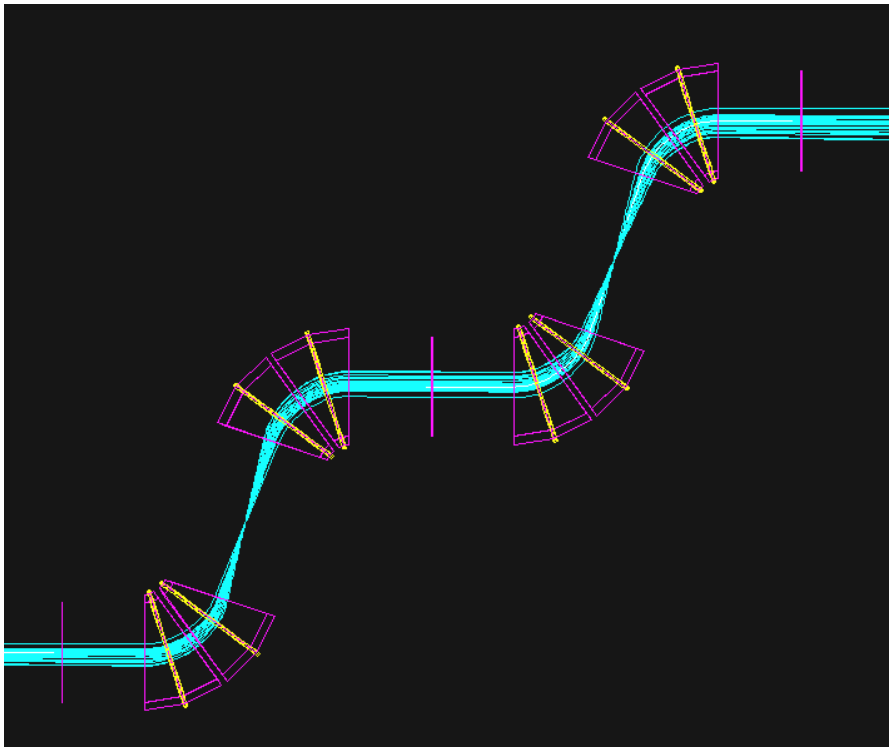
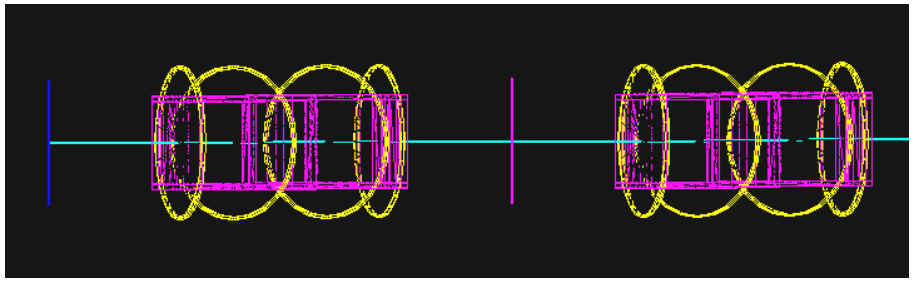
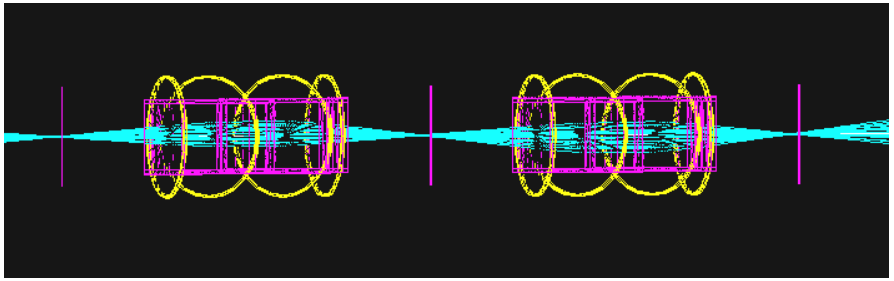




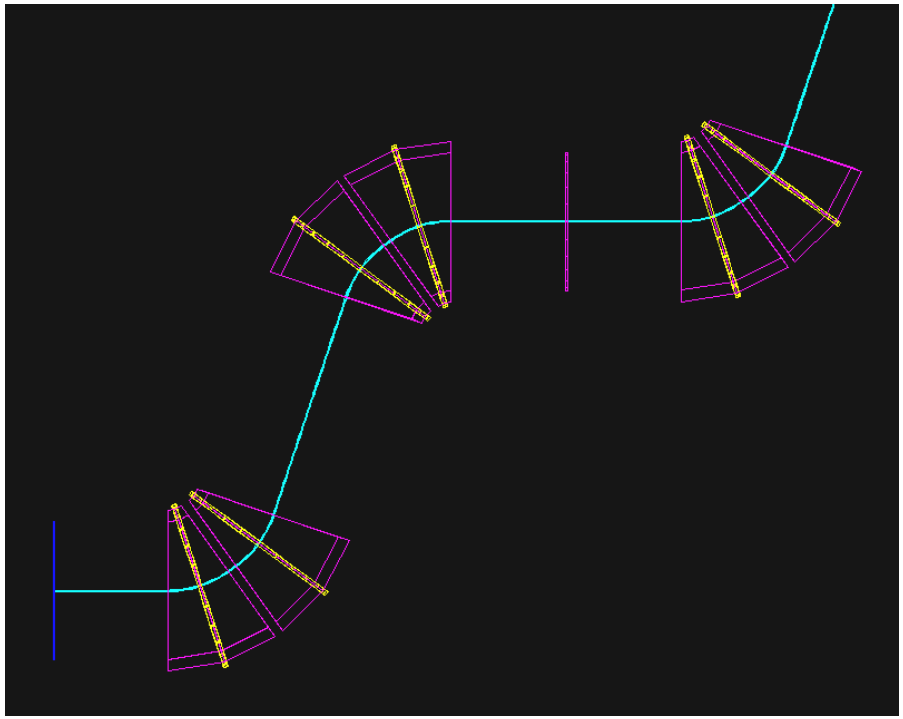
Y

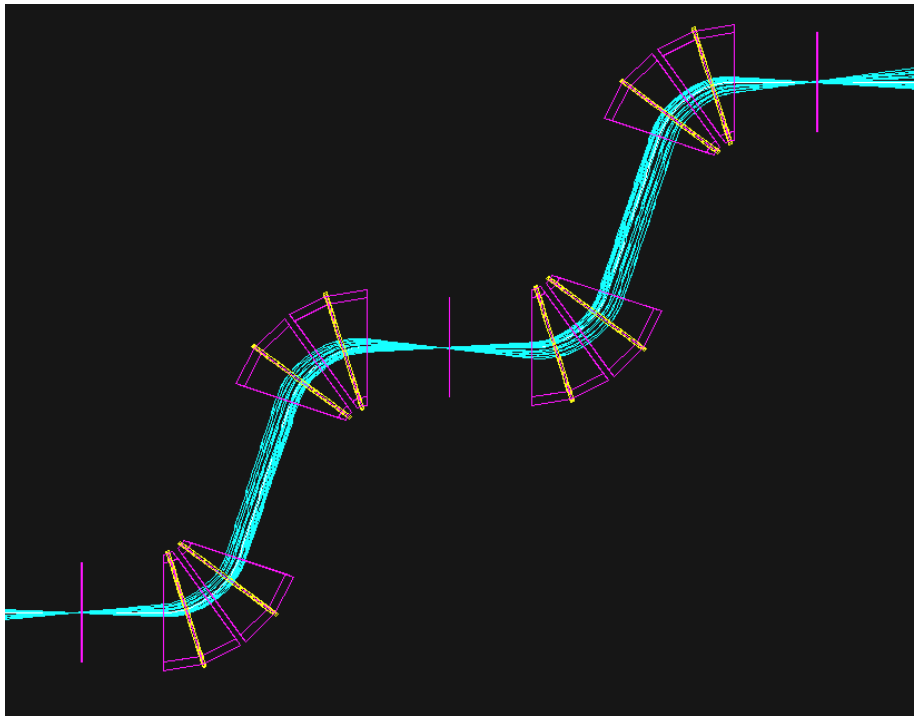


20 Cell (ΔG_Q=-5 T/m)

Cell 1

Cell 19-20





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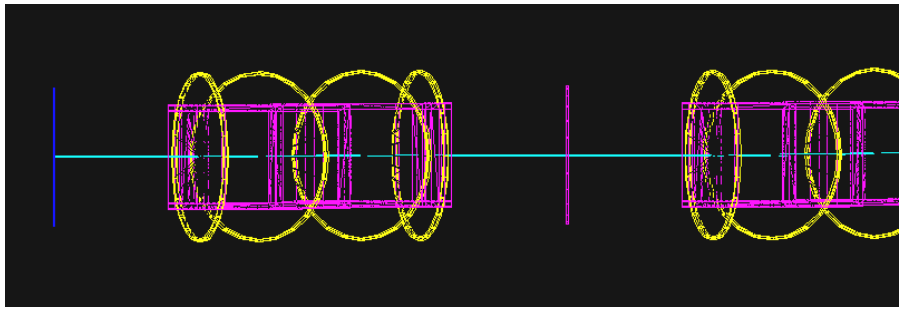
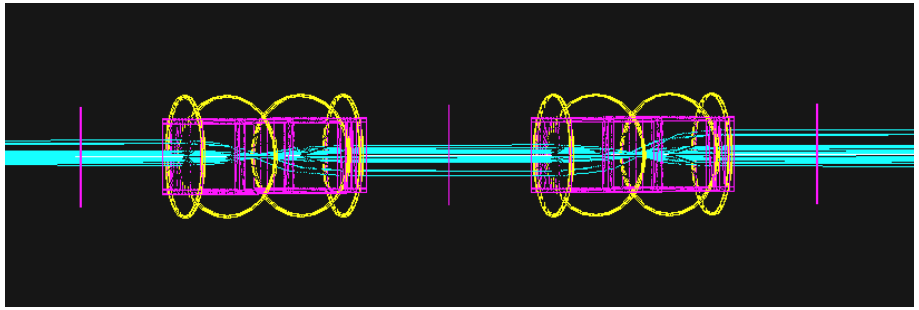
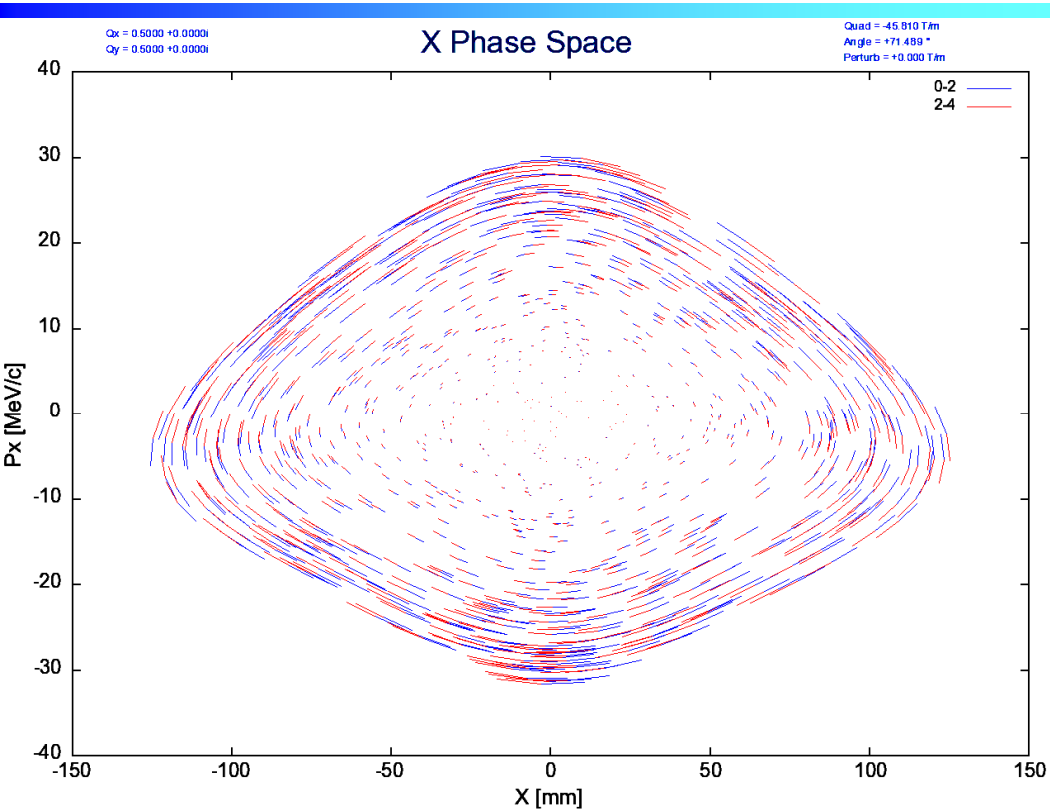


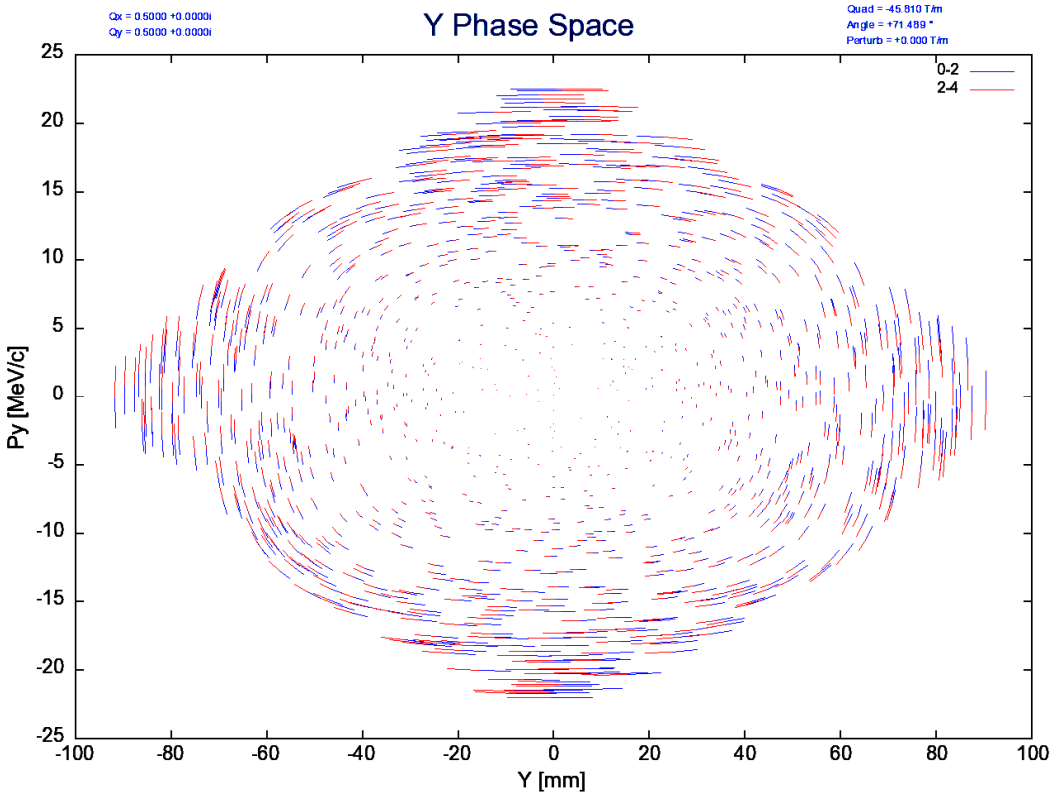


Y

½-Integer Tune Phase Space



- Aberrations are apparent.
- Increased effect with increased amplitude.



G_Q = -45.81 T/m
θ = 71.4887°
Q_x = 0.5000 + 0.0000i
Q_y = 0.5000 + 0.0000i

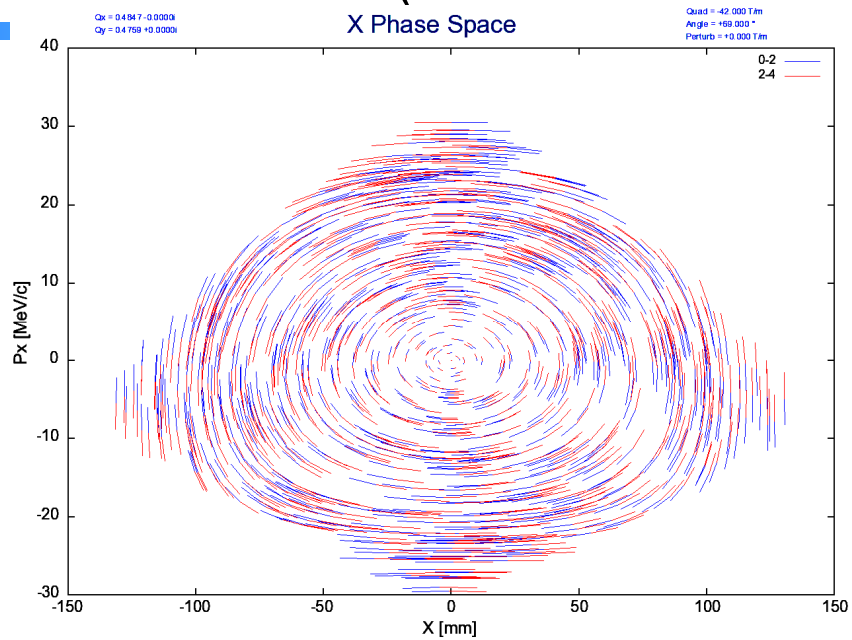
Phase Space Survey

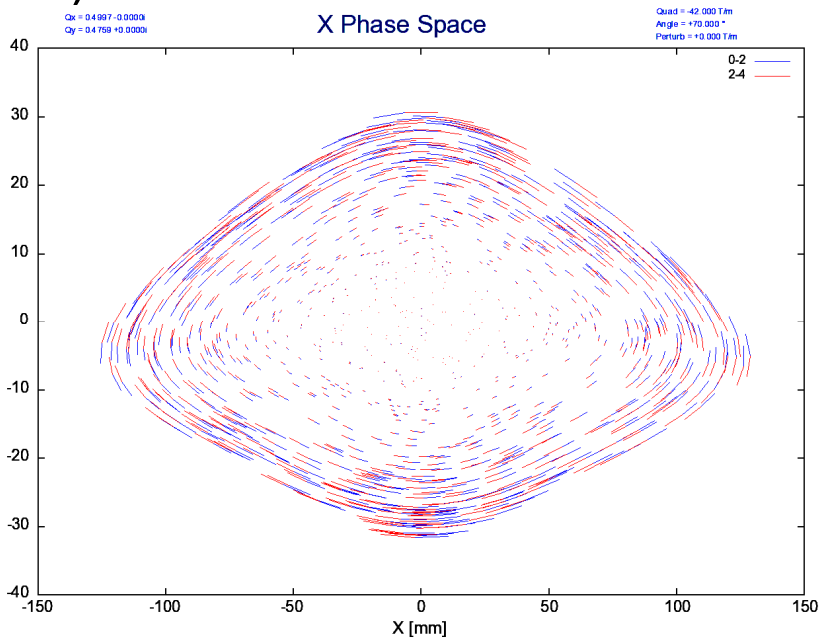
- 2 surveys were performed.
- First was for non-perturbed lattice:
  - G_Q = {-42, -44, -45, -45.81, -47, -48, -50} T/m
  - θ = {69, 70, 71, 71.4887, 72, 73, 74} °
- Second was for a perturbed lattice:
  - G_Q = -45.81 T/m
  - θ = 71.4887 °
  - ΔG_Q = {-1.0, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 1.0} T/m
- Each run generated 2 phase space plots for x and y data calculated independently.
  - X data had all Y and P_y values set to zero and vice versa

Evolution with Increasing Angle

(NO PERTURBATION)

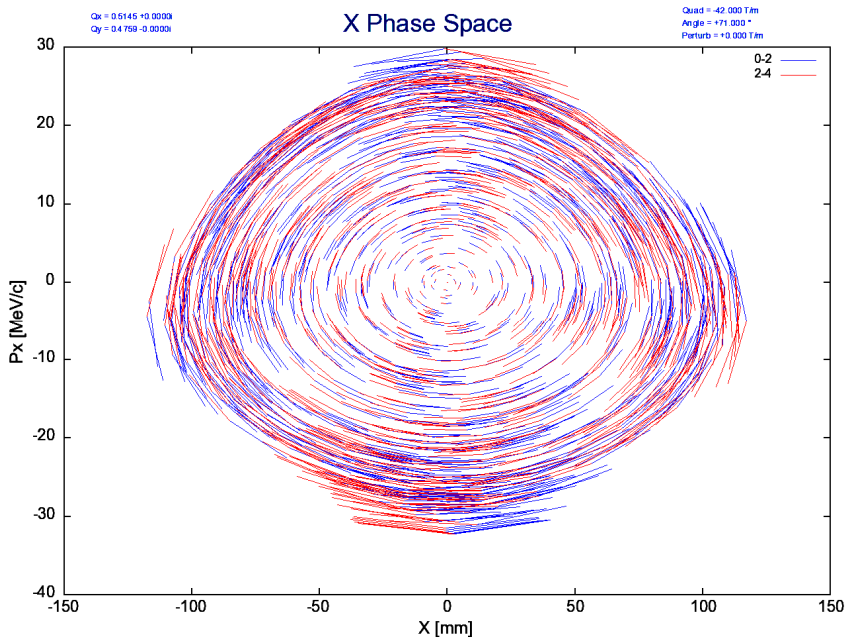
G_Q = -42 T/m
θ = 69°
Q_x = 0.4847 + 0.0i
Q_y = 0.4759 + 0.0i





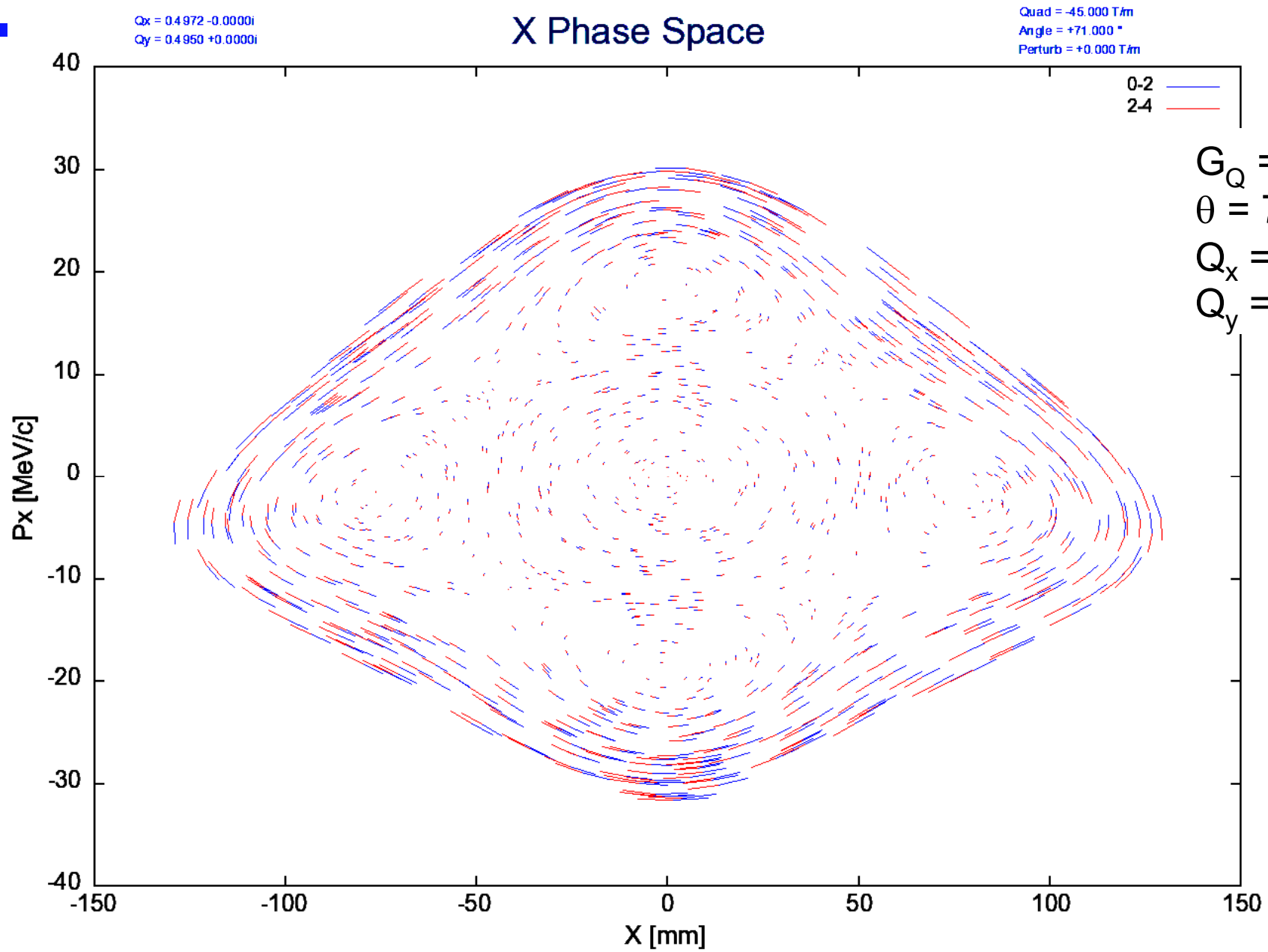
G_Q = -42 T/m
θ = 70°
Q_x = 0.4997 + 0.0i
Q_y = 0.4759 + 0.0i

G_Q = -42 T/m
θ = 71°
Q_x = 0.5145 + 0.0i
Q_y = 0.4759 + 0.0i



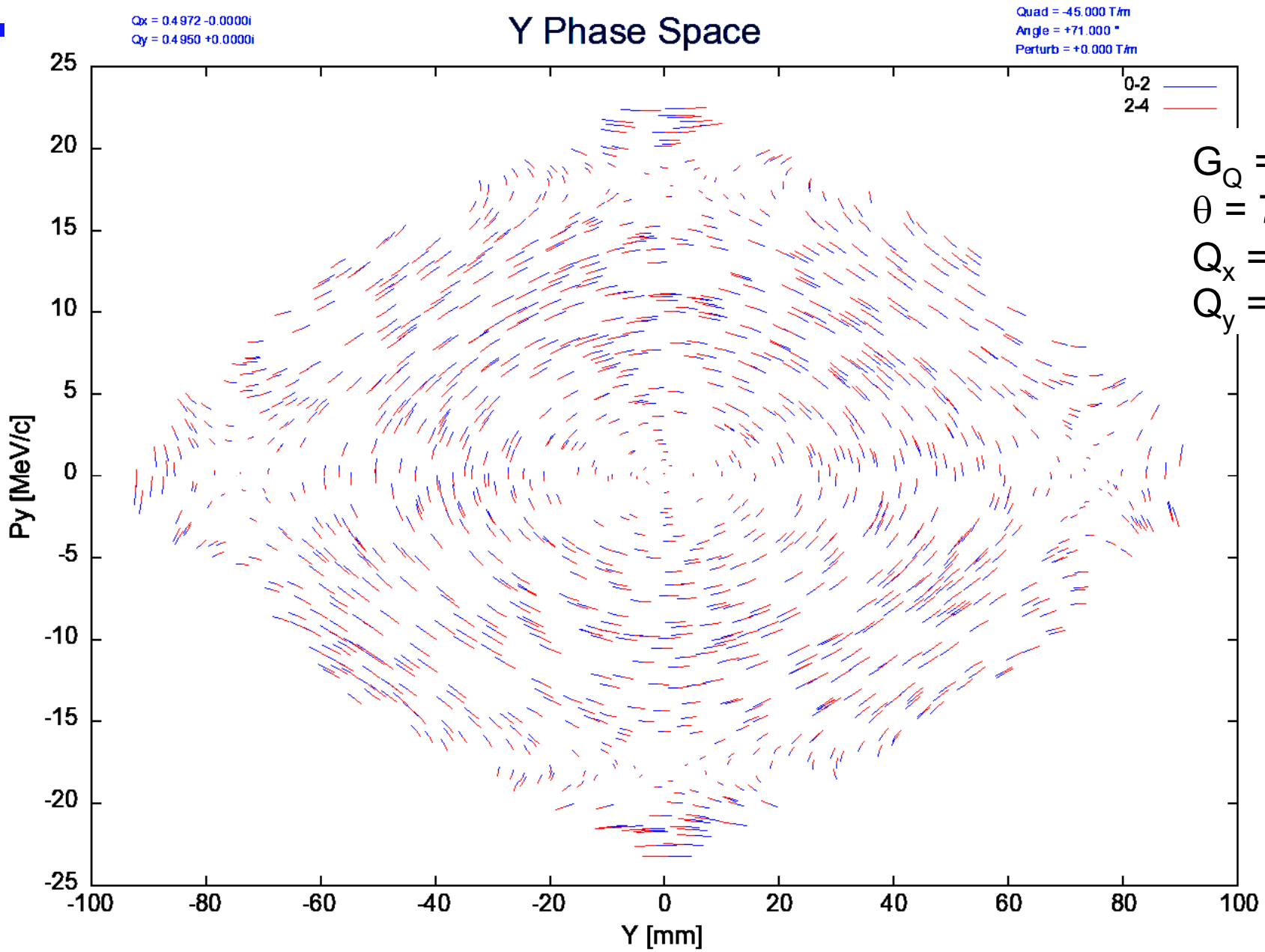
The Y tune does not change. Although not shown, the Y phase space for these three conditions are identical except upon very careful examination.

Interesting Phase Space (X)



G_Q = -45 T/m
θ = 71°
Q_x = 0.4972 + 0.0i
Q_y = 0.4950 + 0.0i

Interesting Phase Space (Y)



G_Q = -45 T/m
θ = 71°
Q_x = 0.4972 + 0.0i
Q_y = 0.4950 + 0.0i

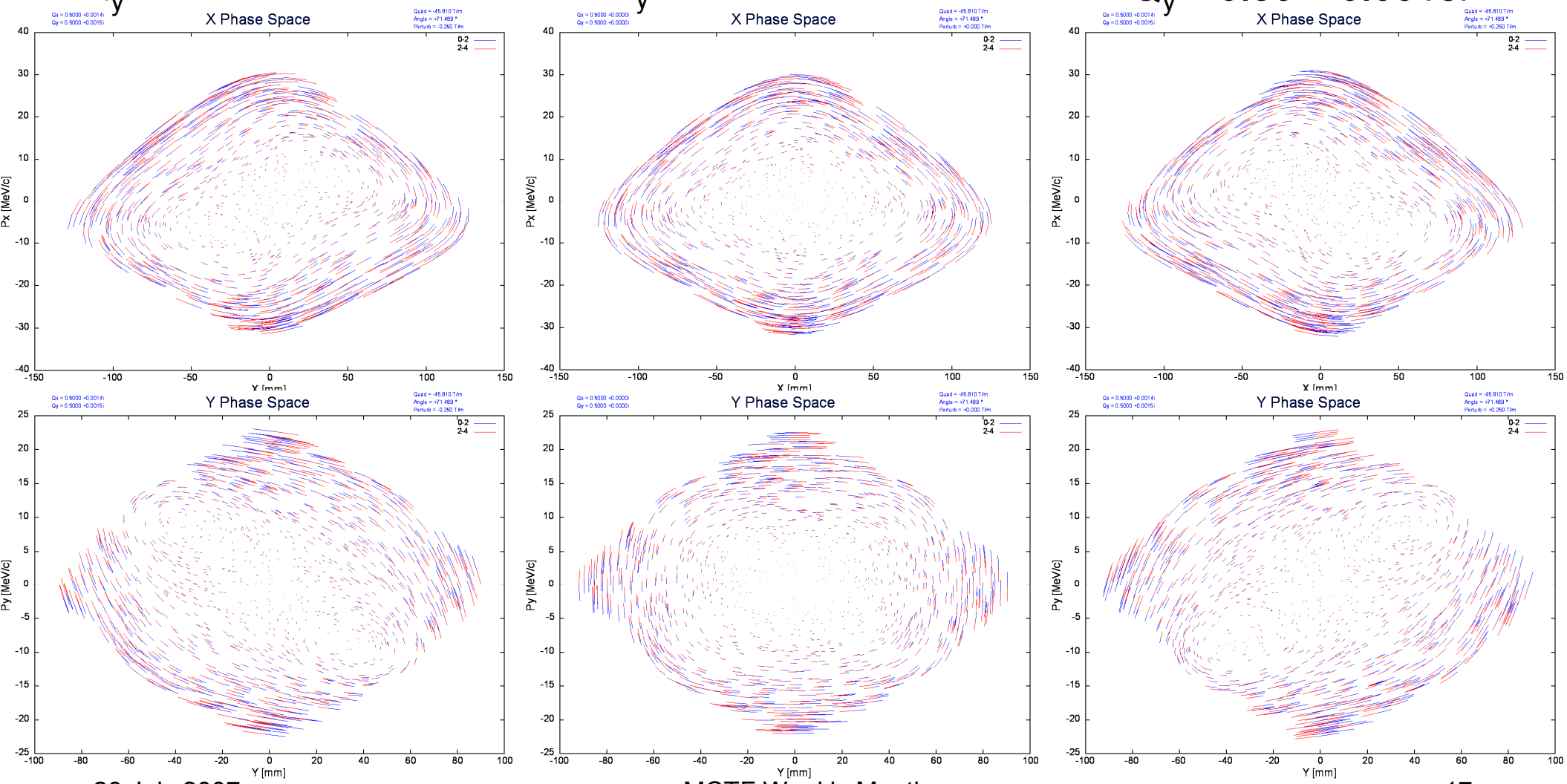
Perturbed on Resonance

("Small" Scale)

ΔG_Q = -0.25 T/m
Q_x = 0.50 + 0.0014i
Q_y = 0.50 + 0.0015i

ΔG_Q = 0.00 T/m
Q_x = 0.50 + 0.0i
Q_y = 0.50 + 0.0i

ΔG_Q = +0.25 T/m
Q_x = 0.50 + 0.0014i
Q_y = 0.50 + 0.0015i



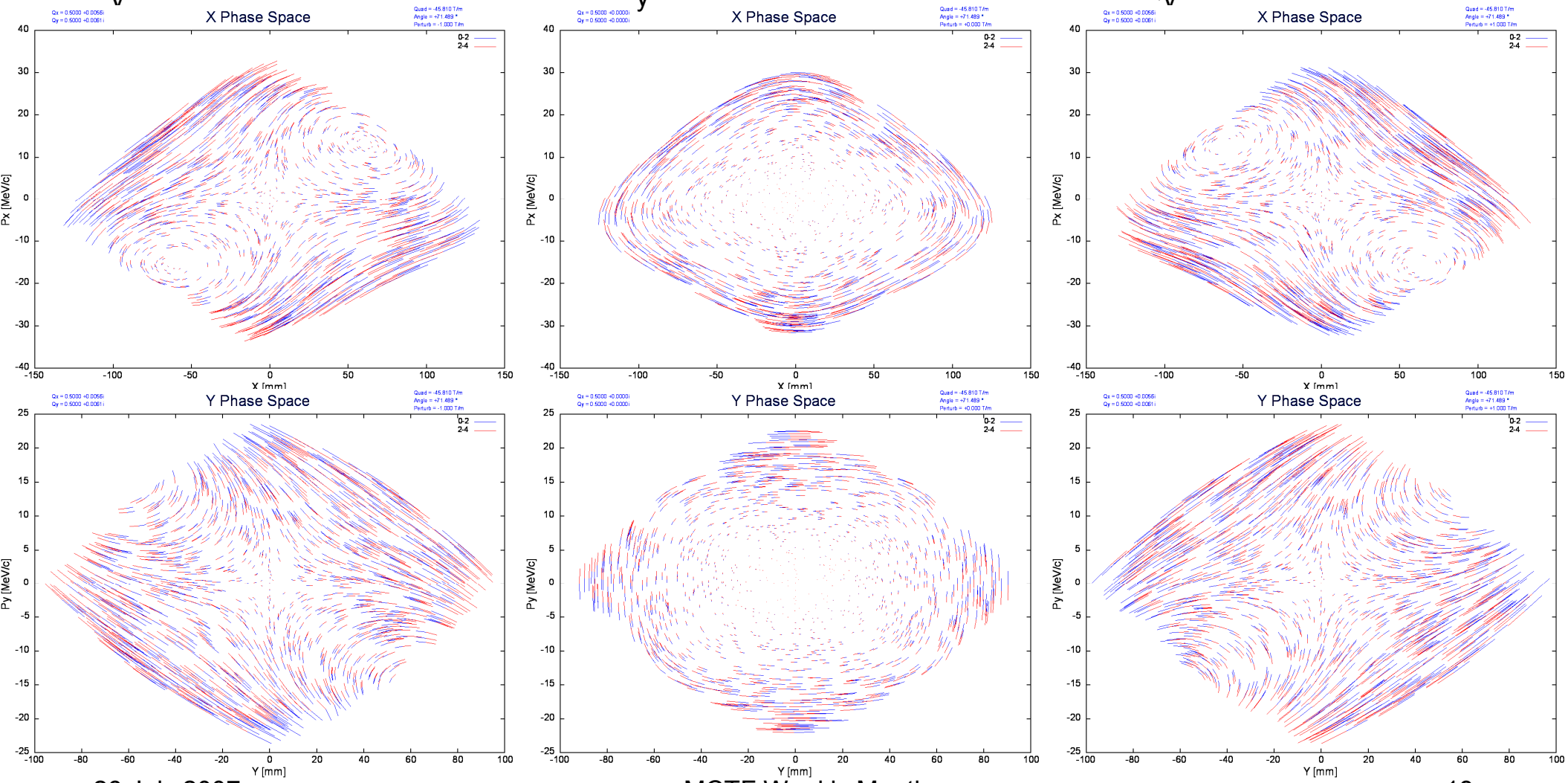
Perturbed on Resonance II

("Large" Scale)

ΔG_Q = -1.0 T/m
Q_x = 0.50 + 0.0056i
Q_y = 0.50 + 0.0061i

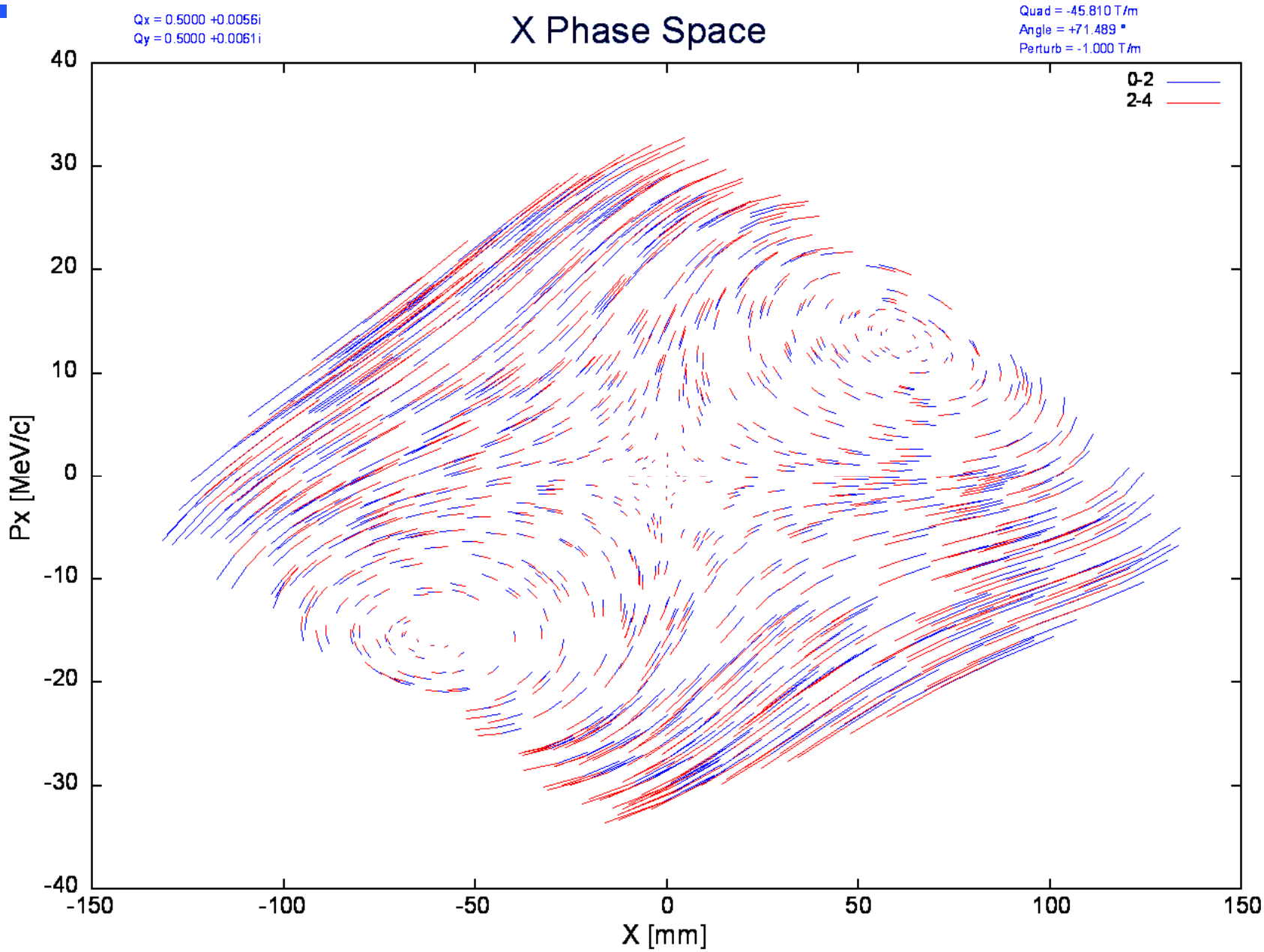
ΔG_Q = 0.00 T/m
Q_x = 0.50 + 0.0i
Q_y = 0.50 + 0.0i

ΔG_Q = +1.0 T/m
Q_x = 0.50 + 0.0056i
Q_y = 0.50 + 0.0061i



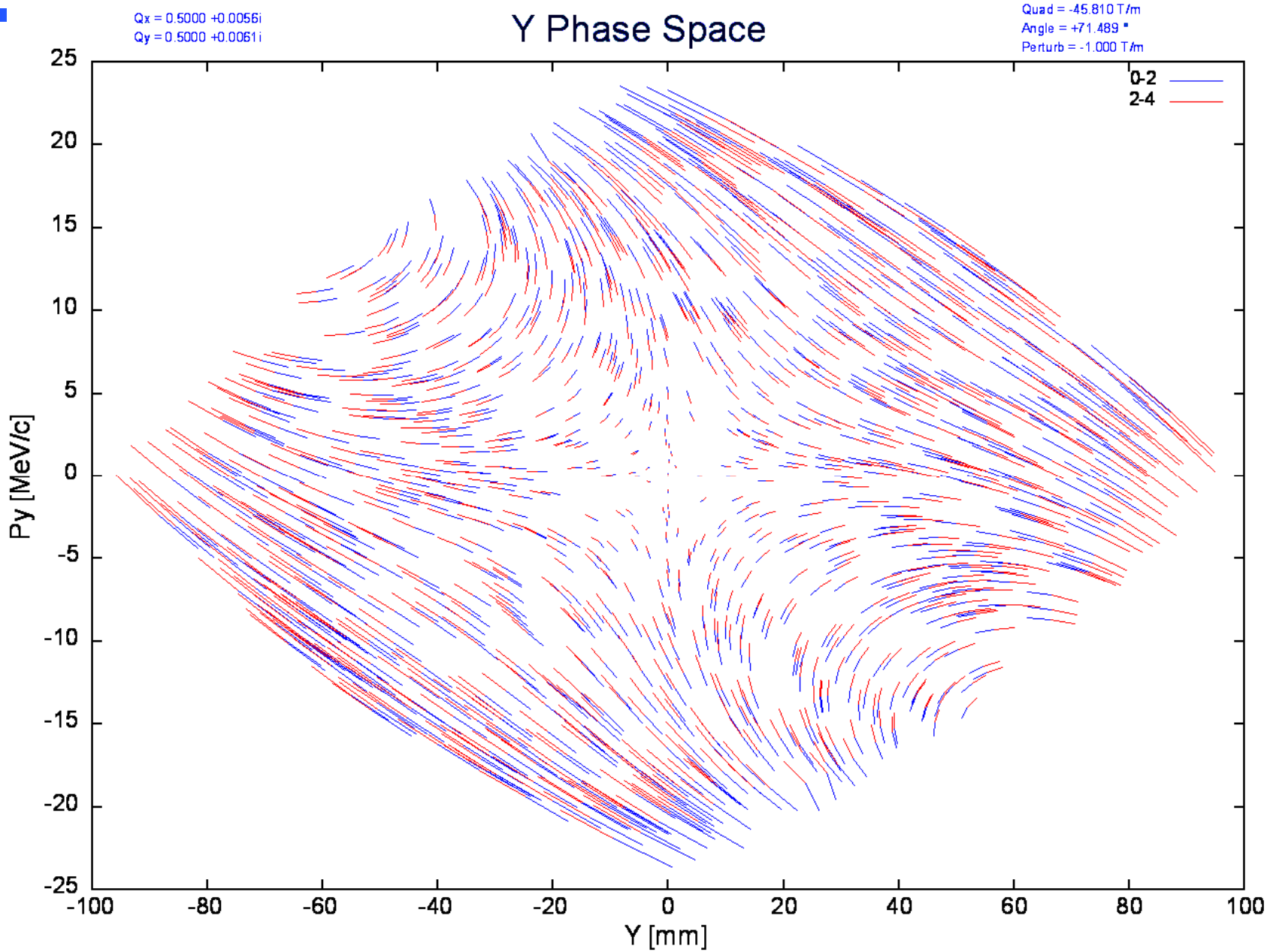
Perturbed Resonance Blow Up (X)

(ΔG_Q = -1 T/m)



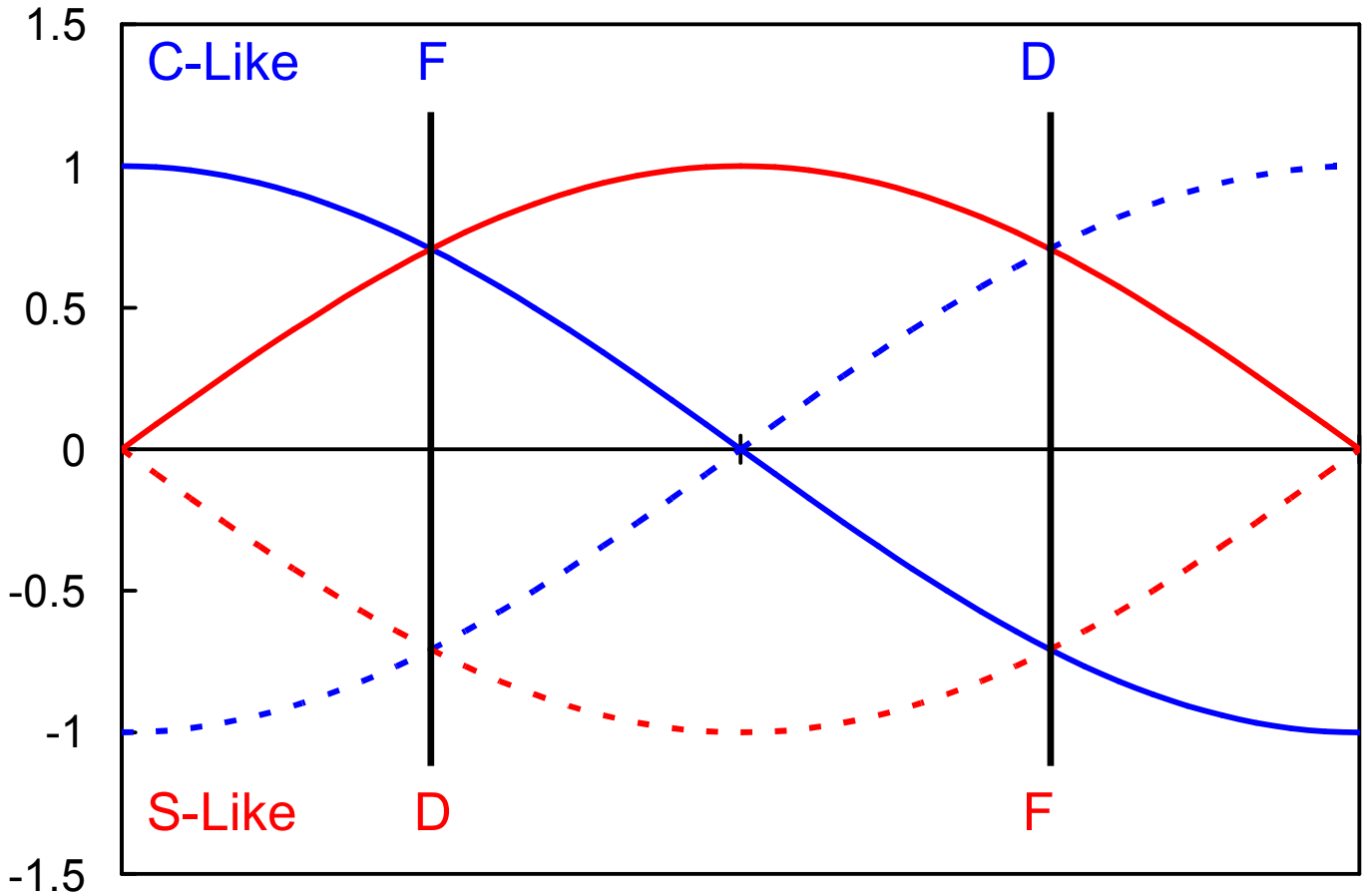
Perturbed Resonance Blow Up (Y)

(ΔG_Q = -1 T/m)



Existing Quadrupole Perturbations

- No change in Q_r because of F-D pair
- Create Sine-Like and Cosine-Like parametric oscillations



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Conclusions from Results

- New PIC lattice appears more stable than the solenoid based lattice.
- Existing quadrupoles can be perturbed to induce parametric resonance with “focal” points at/near symmetry (“absorber”) planes.
- With a simple perturbation of the quadrupole fields, X and Y directions “focus” at planes separated by 90° of betatron phase – symmetric focusing needed.
- The “interesting” phase space tunes *may* provide the opportunity to use the width of the resonance to alleviate some aberrational effects. This must be explored before any firm statement can be made.

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Tune and Fringe Fields

- Simple lattice is attractive
  - No X-Y mixing
  - Trivial independence of tune adjustment
- Addition of fringe fields does
  - Change the elemental focusing in X and Y
  - Complicate the relationship between the independent parameters and the X,Y tunes
- Addition of fringe fields does **not**
  - Produce X-Y mixing
  - Change the linear independence of the parameters-tunes provided the matrix is invertible

$$\begin{pmatrix} Q_x \\ Q_y \end{pmatrix} = \begin{pmatrix} A & B \\ 0 & D \end{pmatrix} \begin{pmatrix} \theta \\ G_Q \end{pmatrix}$$

$$\begin{pmatrix} \theta \\ G_Q \end{pmatrix} = \begin{pmatrix} 1/A & -B/AD \\ 0 & 1/D \end{pmatrix} \begin{pmatrix} Q_x \\ Q_y \end{pmatrix}$$

$$\begin{pmatrix} Q_x \\ Q_y \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \theta \\ G_Q \end{pmatrix}$$

$$\begin{pmatrix} \theta \\ G_Q \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} \begin{pmatrix} Q_x \\ Q_y \end{pmatrix}$$

Future (Current) Directions

- Provide symmetric perturbation
  - Quad doublet
    - Existing quads
    - Additional quads
  - Solenoid
    - End field issues
    - Aberration issues
    - Mixing issues
  - Change θ to compensate the tune in x-direction caused by ΔG_Q
    - Changes dispersion symmetry with a cell
    - May provide slight dispersion offset necessary for PIC
    - “Chicane” layout may add “larger” symmetry
- Study aberration correction
- Include fringe field on elements !!!!