Lattices

R. B. Palmer, R Fernow (BNL)
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INTRODUCTION Cooling & Emit Exchange

Partition functions adjusted by emittance exchange, but $J_x + J_y + J_z = 2$

Equilibrium transverse emittance

$$\epsilon_o = \frac{\beta_\perp}{\beta_v \, J_\perp} \, C(mat, E)$$

Rate of cooling

$$\frac{d\epsilon}{\epsilon} = \frac{dp}{p} \left(1 - \frac{\epsilon}{\epsilon_o}\right)$$

Angular divergence at absorber

$$\sigma_\theta = \sqrt{\left(\frac{\epsilon}{\epsilon_o}\right) \frac{C(mat, E)}{J_\perp \, \beta_v^2 \gamma}}$$

Note conflict on ϵ/ϵ_o

Equilibrium mementum spread

$$\delta_o = \left(\frac{\sigma_p}{p}\right)_o = A(mat) \sqrt{\frac{1}{J_z}}$$

- At minimum of dE/dx ($\approx$ 300 MeV/c)
- For $(\epsilon/\epsilon_o) = 3$
  giving 2/3 of maximum cooling rate
- And $J_x = J_y = J_z = 2/3$

material	$C(mat)_o$ %	A(mat) %	δ_o %	$3\sigma_\theta$ rad
Liquid H_2	0.38	1.66	2.5	0.25
Li	0.69	1.60	2.4	0.35
Be	0.89	1.57	2.4	0.4
C	1.58	1.53	2.3	0.54

- Final rms momentum spread $\approx$ 3%
  So final momentum acceptance $\approx$ 9%
- Angular acceptance very large

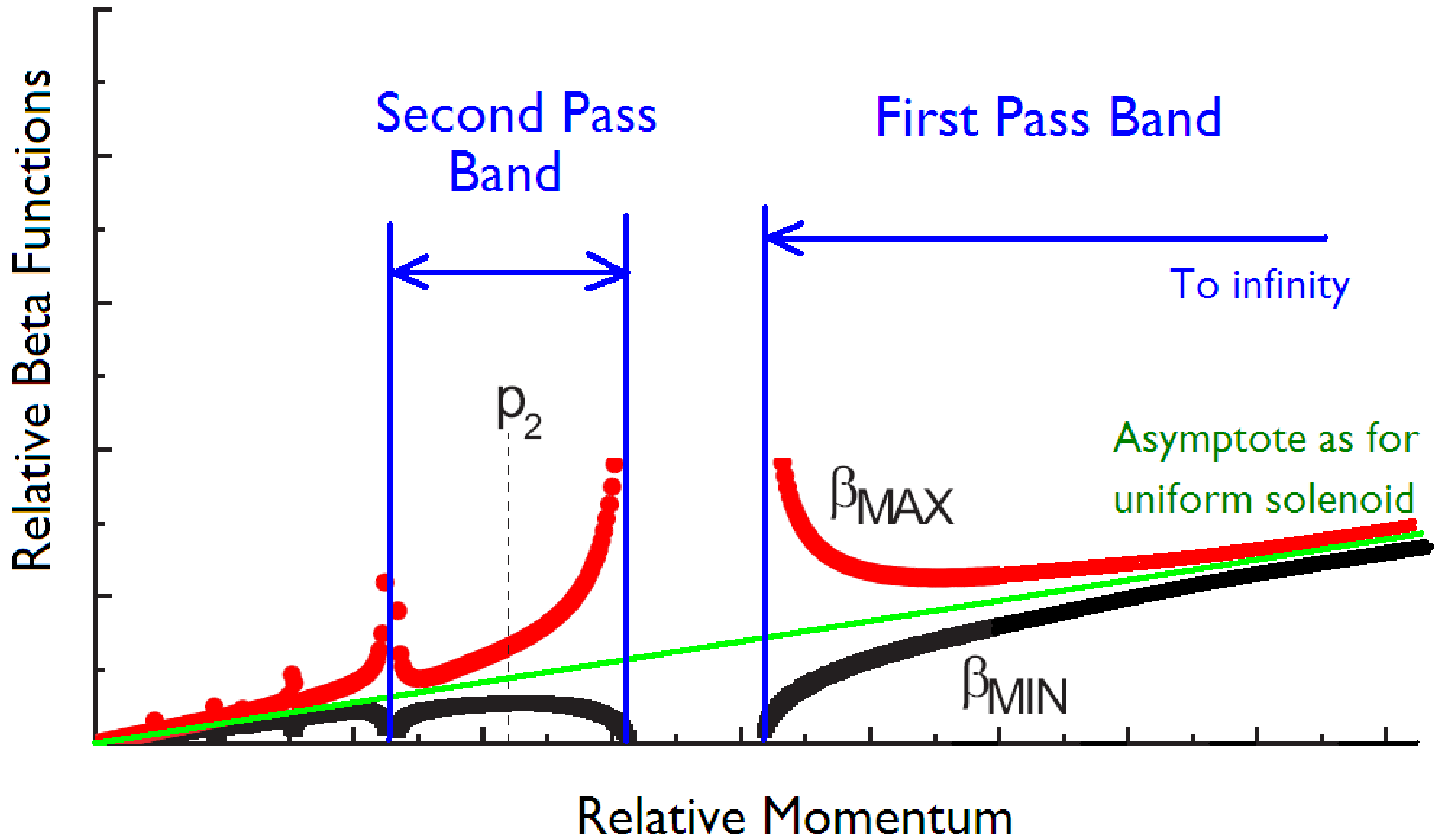
Requirements for Lattice 6D Cooling using Wedges

I will not discuss final linear cooling in 50 T solenoids
nor emittance exchange based on path length differences

- Large transverse angular acceptance 250-350 mrad
- $\beta \propto \epsilon$
  - approximately 30 cm with hydrogen at start for ϵ = 20 mm
  - approximately 3 cm with LiH at end of lattice cooling for ϵ=.4 mm
    in scheme defined at last presentaation using Nb3Sn magnets
  - approximately 1 cm with LiH at end of lattice cooling for ϵ=.17 mm
    may now be possible with HTS magnets
- Large momentum acceptance
  - of order $\pm$ 30 % if longitudinal cooling required (eg early)
  - of order 9 % if only transverse cooling required (eg late)
- Finite dispersion
  - To allow wedges for emittance exchange
- Space for RF without excessive magnetic fields

Pass bands

eg for simple periodic lattice



- First pass band can have unlimmited $\Delta p/p$
- Second pas band has flatter, and lower, beta minimum

Canonical Angular Momentum

Assuming inital average angular momentum ($< r\, p_\phi > = \; 0$), particles entering a solenoidal field cross radial field components and gain angular momentum.
"Canonical" angular momentum, in the absence of material, is conserved.

$$(p_\phi\; r)_{\rm can} \;=\; p_\phi\, r \;+\; \left(\frac{c\; B_z\; r}{2}\right)\; r$$

Without material,when leaving the field, angular momentum returns to its zero canonical value.

Material will reduce all momenta, both longitudinal and transverse. Re-acceleration will not change the angular momenta. The angular momentum will fall. "Canonical" angular momentum gains a negative value On leaving the field, the real angular momentum returns to the canonical value, that is no longer zero.
Further cooling in solenoids of the same sign only makes it worse.

The only solution is to reverse the field, either once, a few, or many times.

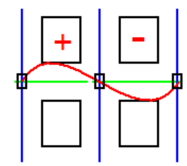
Lattices with alternating fields avoid the difficult problem of designing the otherwise needed "achromatic field reversals".

A Lattice type notation

We will be considering only axial lattices formed of a number of axially symmetric coils

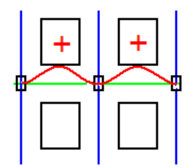
To distinguish different lattices we will use a simple notation indicating the signs of the coil currents in each solenoid for two geometric cells. Consider a lattice with one magnets per geometric cell and an absorber location between the cells. If the alternate cells have opposite polarities and the lattice is used in its first (highest momentum) pass band, then the notation is

$$< +|- >_1$$



If the two geometric cells have the same polarities then the notation is

$$< +|+ >_1$$

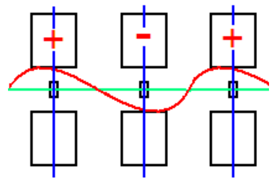


Note the use of the term "geometric cell". Since the focusing in an axial solenoid system depends only on the square of the magnetic field, both the orbits in the above lattices have the same symmetry, both consisting of two identical "geometric cells". Only if bending is introduced, however small, is this symmetry lost for $< +|- >$, but maintained for $< +|+ >$.

Lattice Notation continued

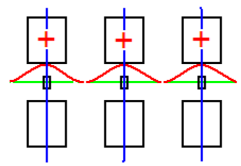
Solutions also exist where the beta minima are under one of the coils. For example, alttace with alternating coils, operating in the second pass band, the notations are:

$$< \hat{+}\hat{-}\hat{+} >_2$$



and if they are not alternating:

$$< \hat{+}\hat{+}\hat{+} >_2$$



Here the check mark replaces the bar to indicate the absorber location. By default, the polarity of two cells, plus two half magnets, are now indicated. The number of signs is thus alway odd.

Scaling β and all lengths, with p and a B_{max}

A given geometry, will have a given $\Delta p/p$ independent of scale but can have different scales in length L.

$$\beta \propto L$$

which will scale as p/B. In particular, we can scale β by the β_o in a long solenoid with field equal to the maximum field on axis in our lattice.

Using units for p in electron Volts, and $e = \pm c$ in MKS:

$$\beta_o = \frac{2p}{eB_{max}}$$

So we can define a "normalized" β, F_1, as a function of the geometry, independent of scale:

$$F_1 = \frac{\beta}{\beta_o} = \frac{\beta c B_{max}}{2\ p}$$

Scaling with a maximum current density J_{max}

If HTS materials are used there is effectively no constraint on B, but a significant constraint on the current density J

From above, with units such that $e = \pm c$in MKS we have

$$B \propto c\,\frac{p}{L} \quad \text{and} \quad B \ \propto \ \mu_o\,\frac{I}{L}$$

so

$$I \propto \left(\frac{\mu_o}{e}\right) p$$

independent of L. The current densities J including the maximum J_{max} scale as

$$J \ \propto \ I/L^2$$

so

$$\beta \ \propto \ L \ \propto \ \sqrt{\frac{p}{\mu_o e J_{max}}}$$

And we can define a second "normalized" β, F_2:

$$F_2 \ = \ \frac{\beta}{\sqrt{\frac{p}{J_{max}\mu_o e}}}$$

Obtaining βs for given attainable B, or J

We will determine and plot F_1 and F_2 values for differing lattices, as a function of their momentum acceptance $\Delta p/p$.

Then, for a required $\Delta p/p$, and given attainable B_{max}, or J_{max}:

$$\beta = F_1 \left(\frac{2p}{eB_{max}}\right)$$

or

$$\beta = F_2 \sqrt{\frac{p}{J_{max}\mu_o e}}$$

again with $e = \pm c$ in MKS

LATTICES

1) A CONTINUOUS SOLENOID

The β for momentum p in a continuous solenoid with p in electron Volts:

$$\beta = \frac{2\,p}{c\,B}$$

So at the momentum p:

$$F_1 = 1$$

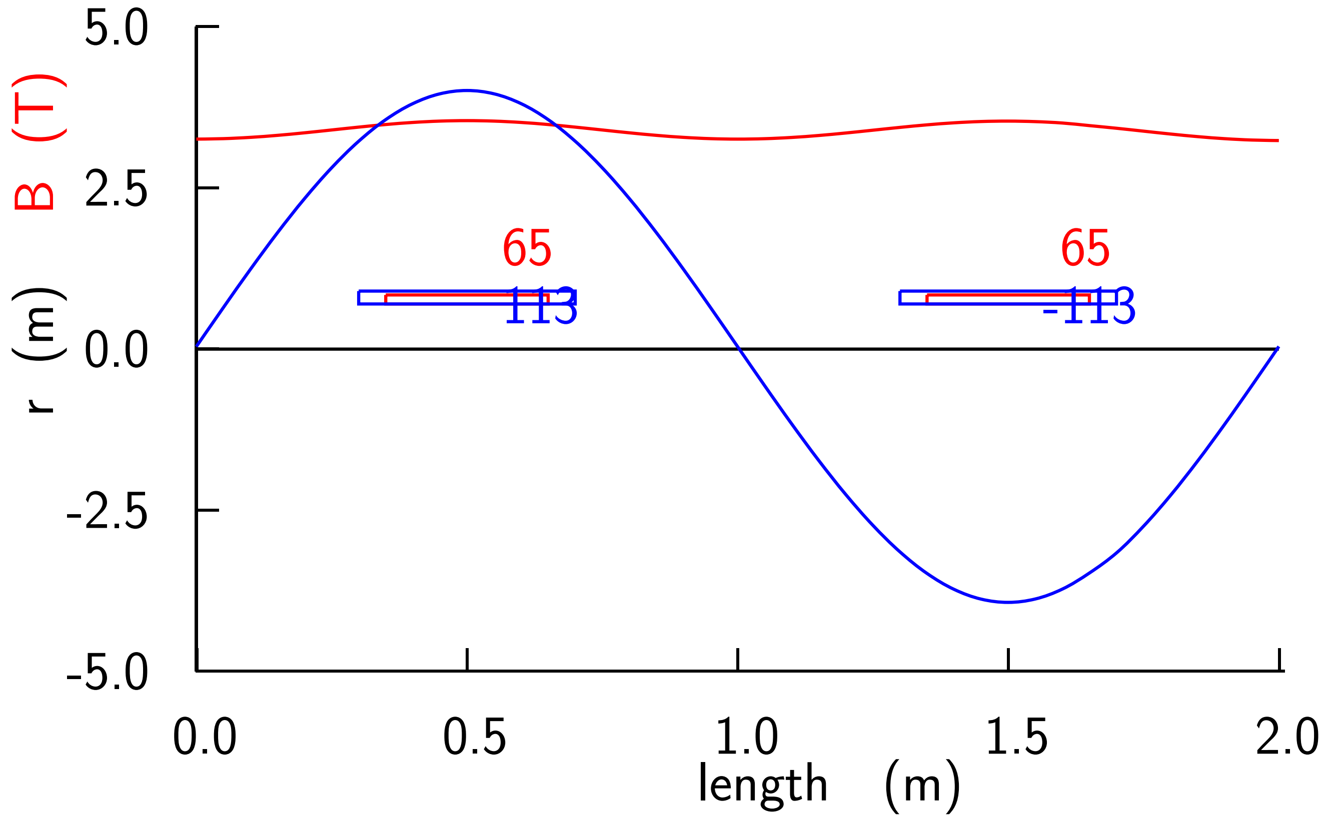
For a finite momentum acceptance $\Delta p/p$, the maximum beta will be

$$F_1(\mathrm{max}) = \left(1 + \frac{\Delta \mathrm{p}}{\mathrm{p}}\right)$$

2) ONE COIL PER CELL IN FIRST PASS BAND

"FOFO" with $< +|- >_1$, or without field reversals $< +|+ >_1$

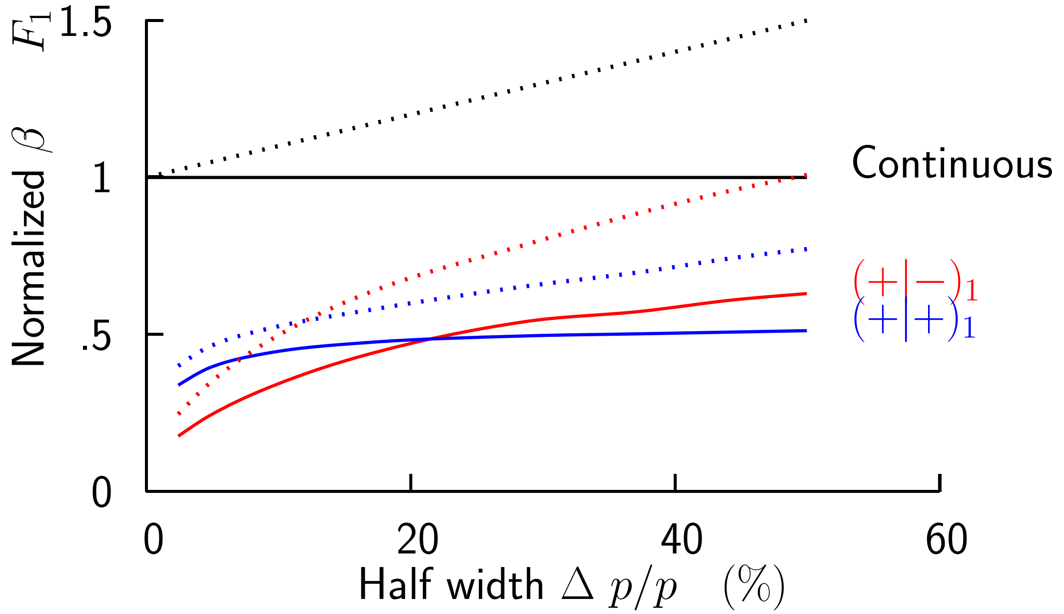
$< +|- >_1$ was used in Feasibility Study 1, and $< +|+ >_1$ was considered as an alternative



Non-scaled parameters of all cases are given in the appendices

Normalized Betas vs. half width $\Delta p/p$

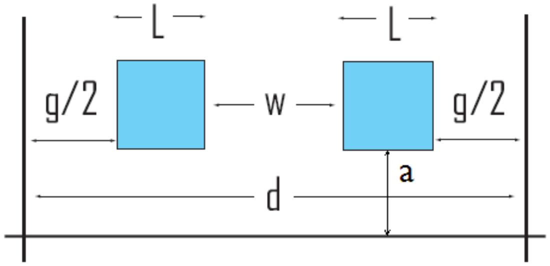
- $\Delta p/p$ varied by choice of operating distance from start of stop band
- Line represents beta at center of band   Dots are for maximum betas



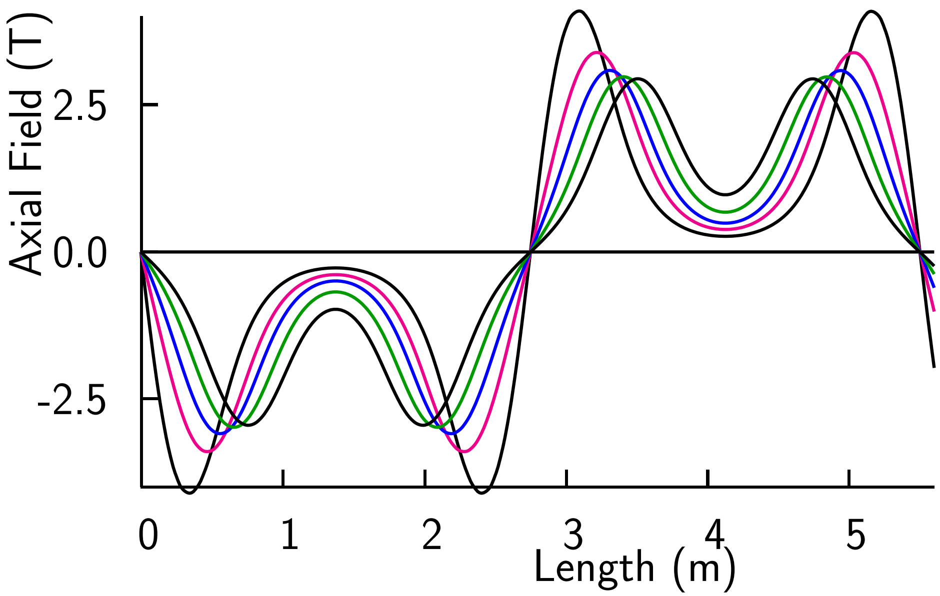
- Cases with and without reversals give lower betas than a continuous solenoid
- At low $\Delta p/p$ With reversals has an advantage
- At larger $\Delta p/p$ without reversals has advantage
  But without reversals requires eventual reversal to resore canonical angular momentum

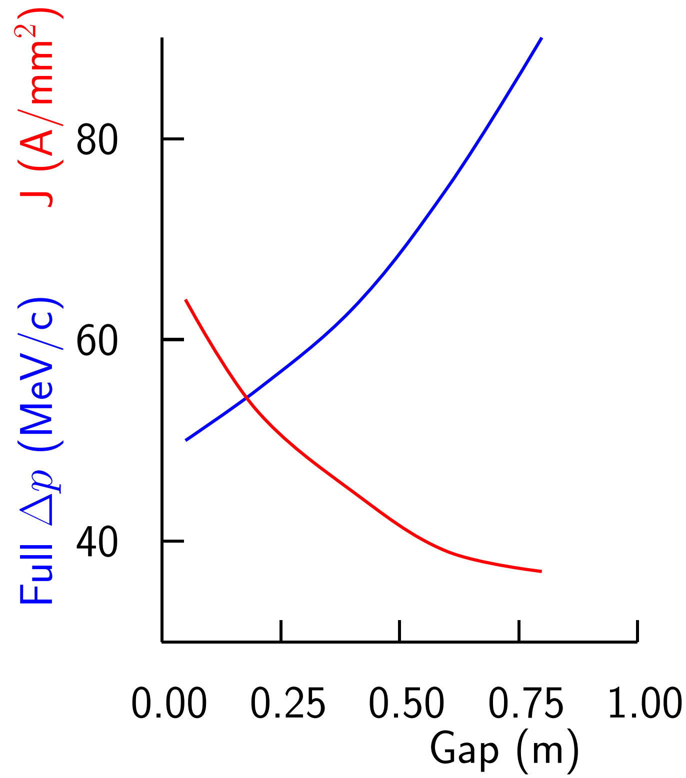
3) TWO COILS PER CELL IN 2ND PASS BAND

Original "Super FOFO" (Sessler) $< + + | - - >_2$

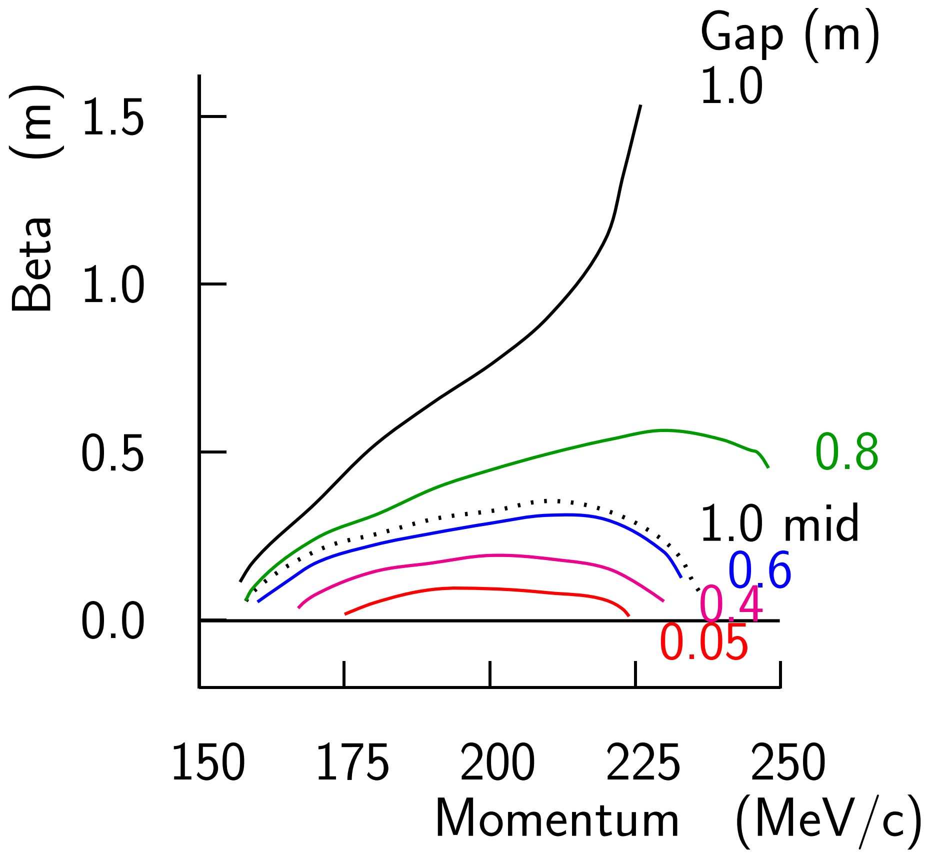


- g is varied to vary $\Delta p/p$
- J set to center mom acceptance



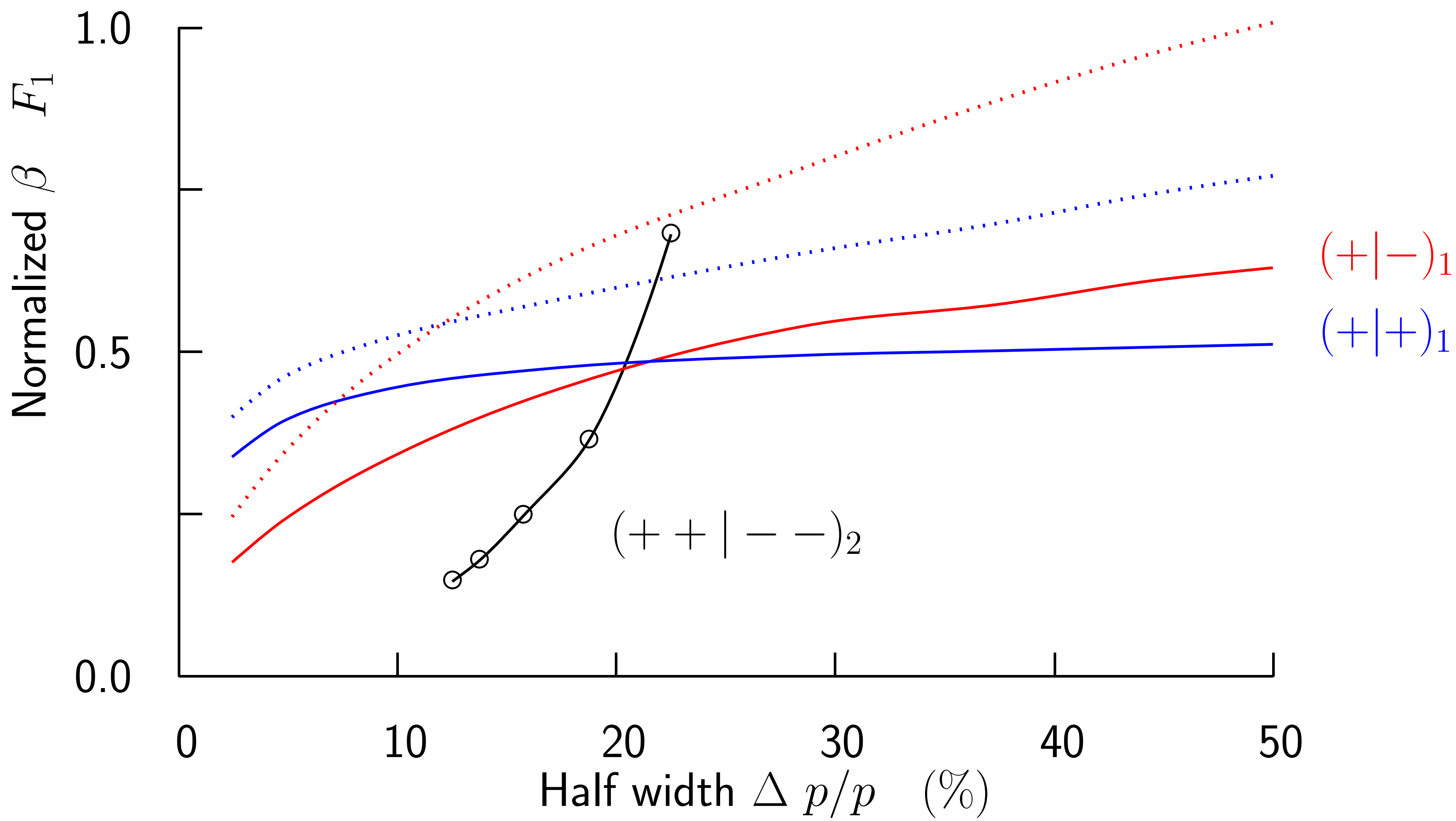


Betas vs. Momentum



- Note the "bad" beta shape for g=1.0
  Occurs when mid-cell focusing is too weak. Found also when a is increased
- Minimum "good" betas are now in mid point of cell (see dotted)
  i.e. Symmetry is now $< + - | - + >_2$ More of this later

Normalized Betas vs. half width $\Delta p/p$



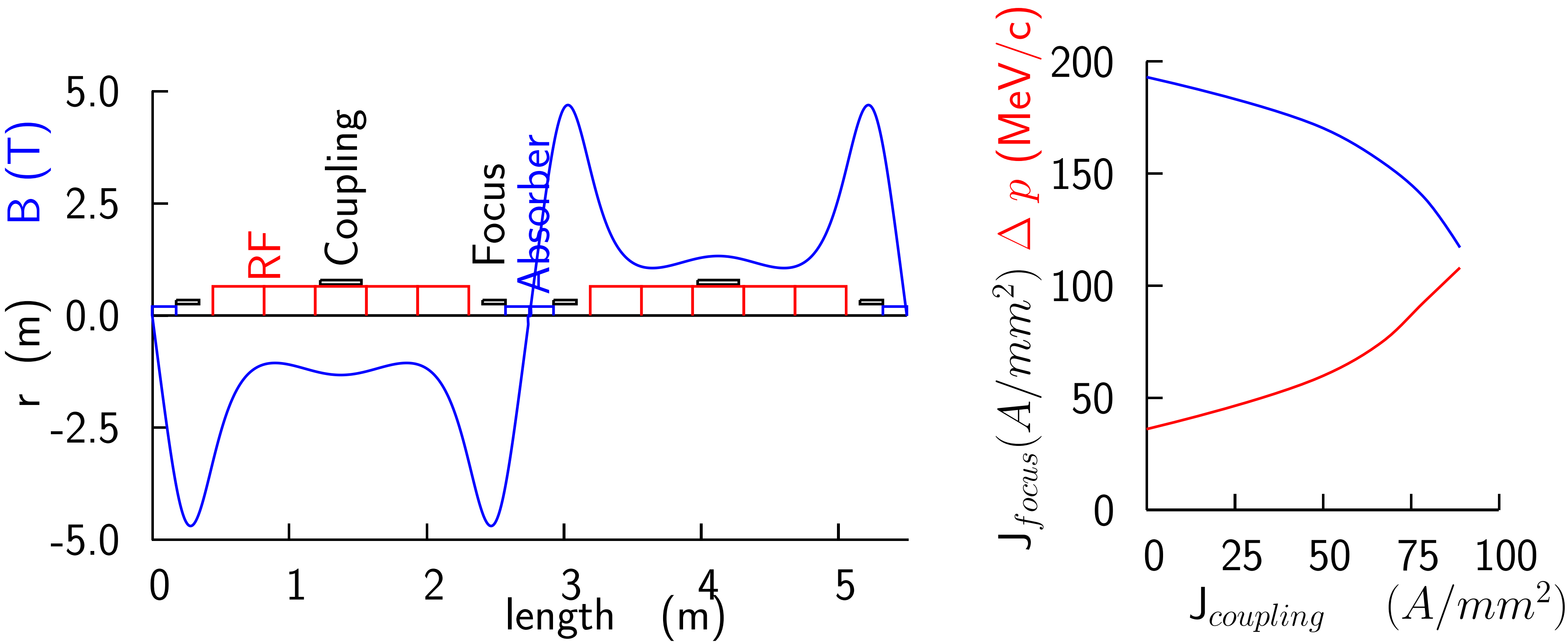
- For $\Delta p/p > 22$ % FOFO lattices in first pass band prefered
  In early cooling, for instance
- But for $\Delta p/p < 22$% SFOFO Lattices in 2nd pass band are prefered
  For instance, after emittance exchange to reduce $\Delta p/p$

Problems with 2 coil SFOFO

1. With the Study 2 parameters, the coils were too small to enclose a 200 MHz RF cavity
   Increasing their radius causes the beta shape to become bad
2. Cooling is best if the beta is continuously reduced as $\epsilon_\perp$ falls
   but changing the geometry at every cell is expensive
   A "tunable lattice would be prefered

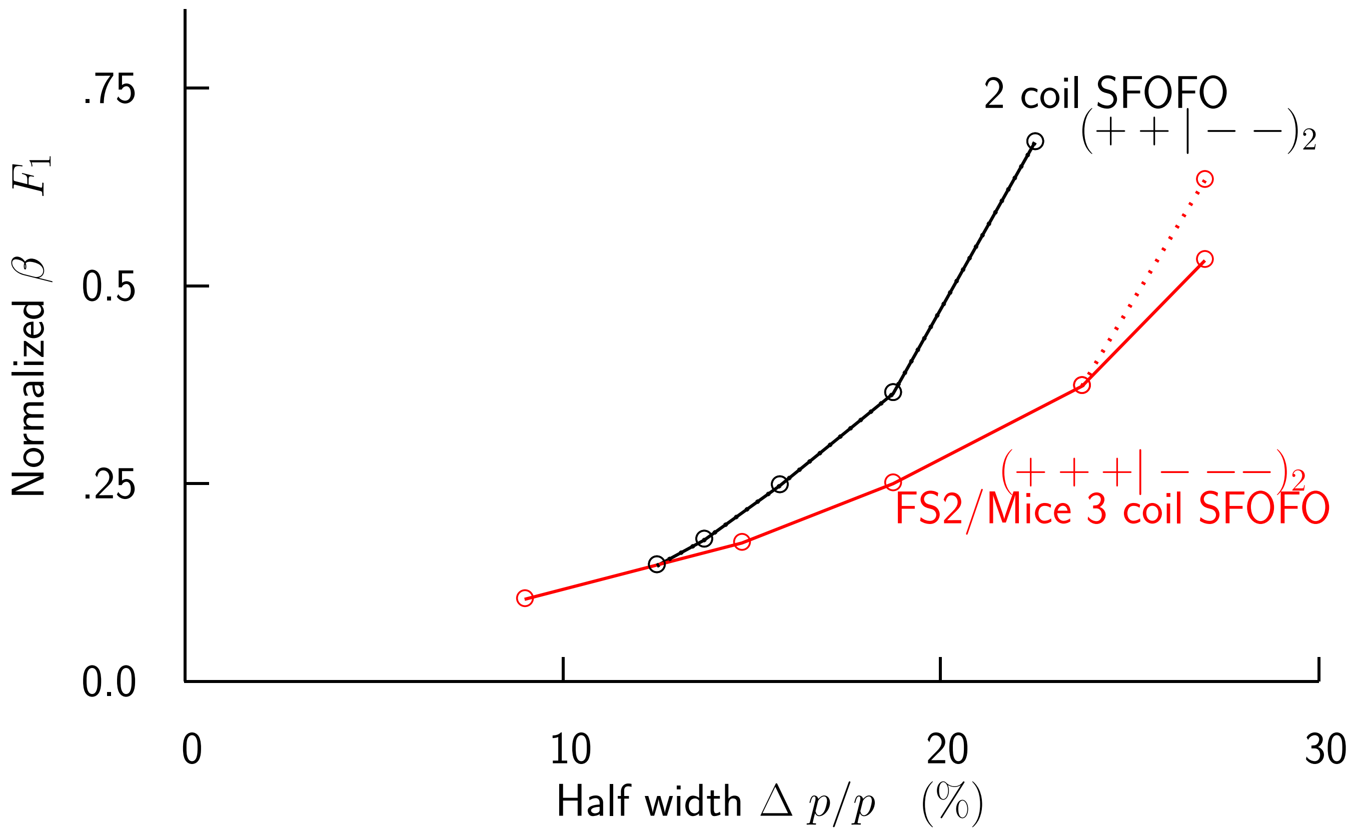
4) THREE COILS PER CELL 2ND PASS BAND

In Study 2, and in MICE $< + + + | - - - >_2$



- "Focus" coils are close in to the absorber
- "Coupling" coils are outside the RF
- Decreasing the "coupling" currents lowers $\Delta p/p$ and β^* at the absorbers
- Re-adjust focus current to center momentum acceptance

Normalized β

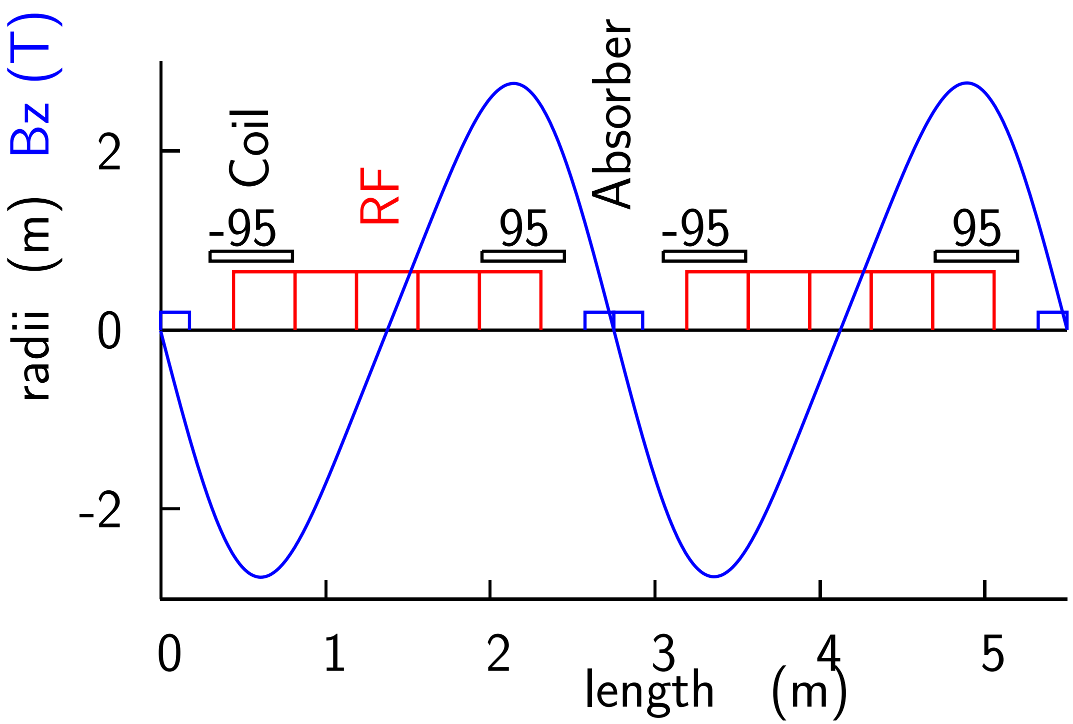


- 3 coil SFOFO is better at moderate $\Delta p/p$
- But difference goes away as $J_{coupling}$ is reduced
  Indeed the last point with $J_{coupling} = 0$ is really 2 coil

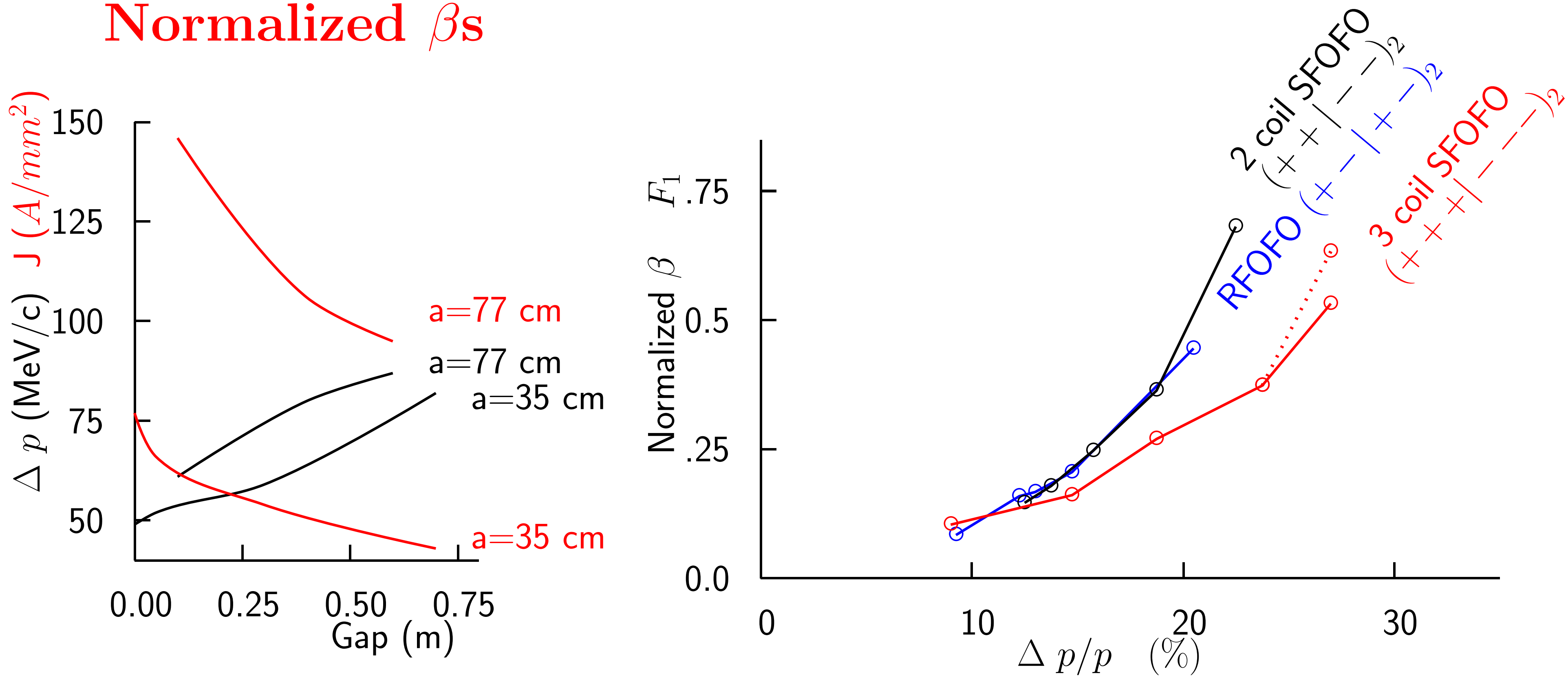
5) TWO COILS PER CELL, NON-ALTERNATING

RFOFO for Emittance Exchange Rings $< + - | + - >_2$

- When bending is introduced dynamics depend on the signs of the cell
- An SFOFO lattice would now have double the number of resoncaces including one in the middle of the "2nd pass band"
- We thus look for a lattice with all cells truely alike, but with zero integral B
- This lattice was used in our cooling rings with wedge emittance exchange



Normalized βs

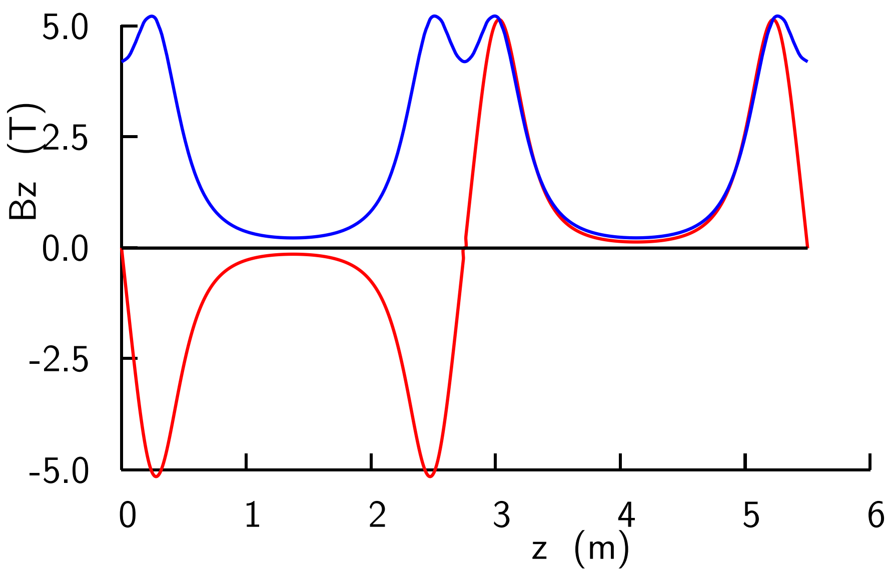


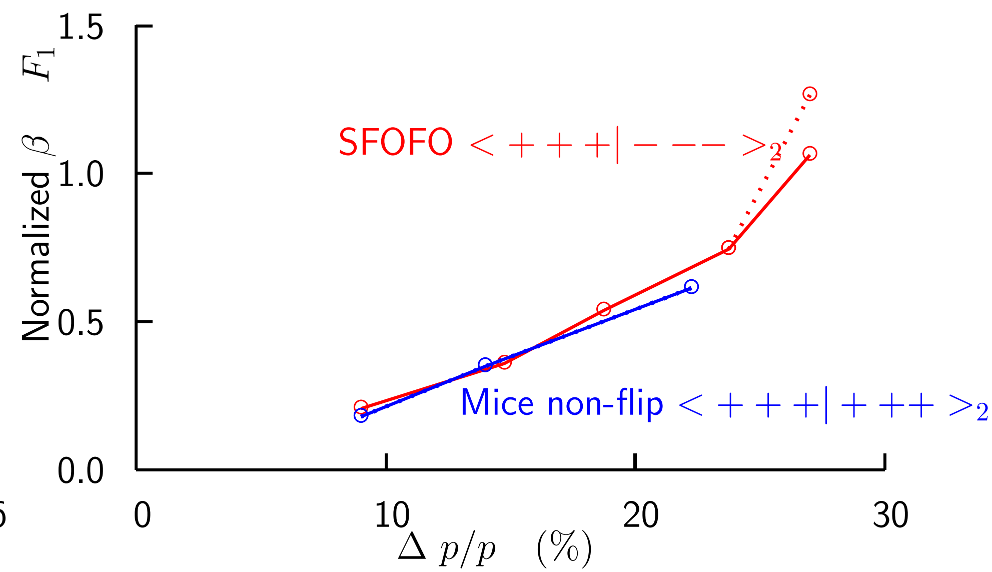
- $\Delta p/p$ and Normalized βs not sensetive to coil radii
  because the fields on axis are similar
- But current densities required are much higher for larger radii
- Performance in either case is simular to SFOFO lattices

6) "NON-FLIP" LATTICES

We use the term "flip" to indicate when the field reverses across the absorber giving a zero field at the absorber center. Non=flip lattices have a finite field over the entire absorber

- Compare 3 coil SFOFO solutions with and without flip

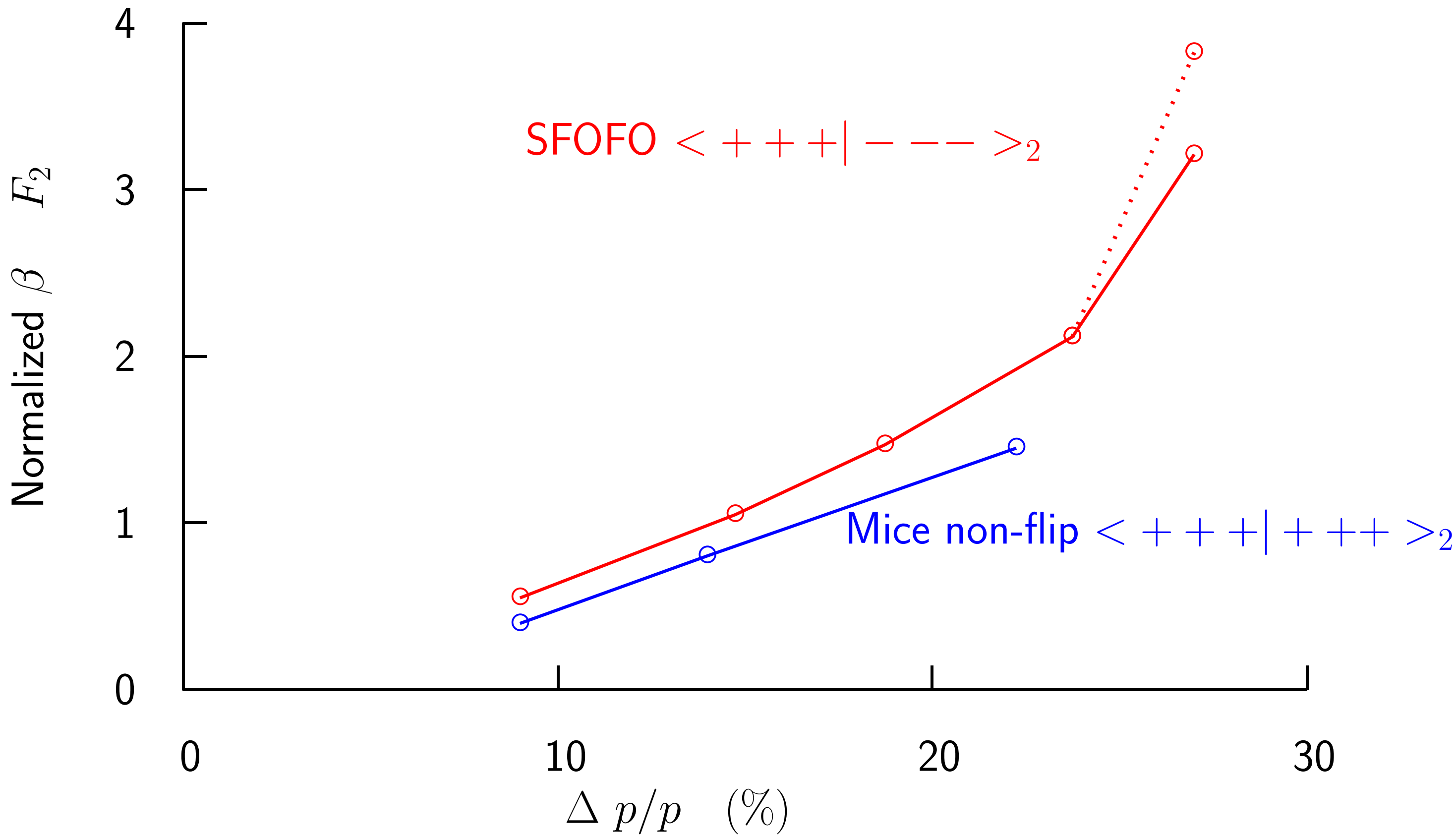




- Both seem to work equally well
- βs, $B(max)s$ and F_1s are almost identical
- But Current densities are less
  eg for minimum beta cases:

	Flip	Non-flip
J (A/mm^2	193	149

Look at Second Normalized β: F_2

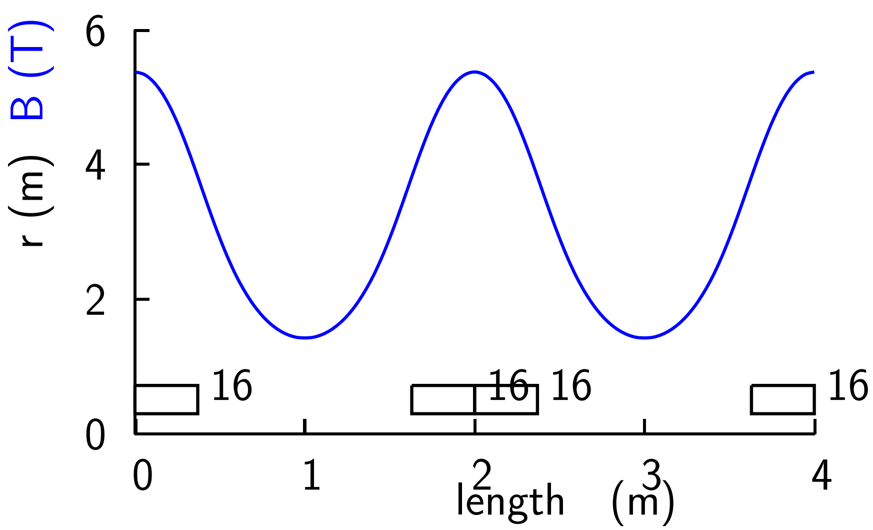


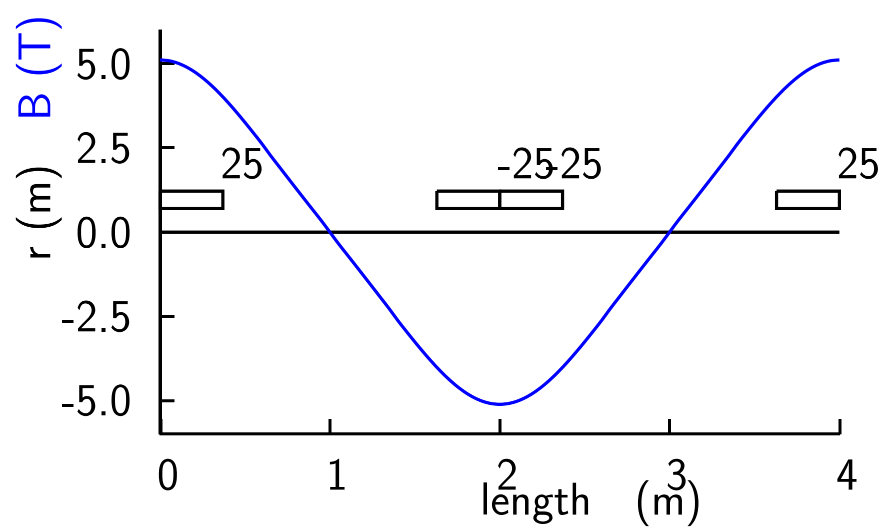
- But if Field does not reverse, the coils can extend to larger radii
- And can be continuous across the minimum beta
- This will lower Js and F_2s further

7) SINGLE COIL CELLS IN 2nd PASS BAND

Fernow's new $< \hat{+}\hat{+}\hat{+} >_2$ and $< \hat{+}\hat{-}\hat{+} >_2$

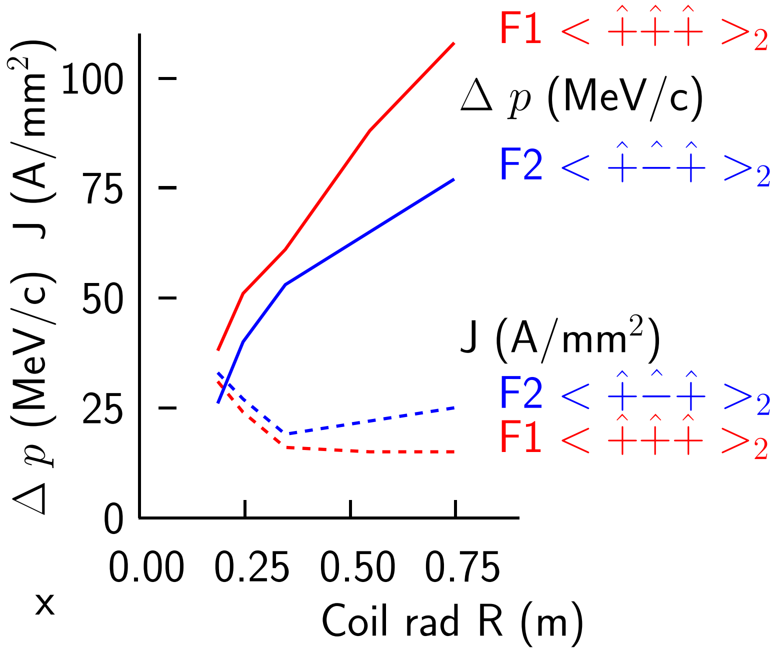
- Both are "non-flip" solutions with finite field on the absorber
- F1 has single non-alternating coils used in the 2nd band: $< \hat{+}\hat{+}\hat{+} >_2$
- F2 has single alternating coils in the second band : $< \hat{+}\hat{-}\hat{+} >_2$
  - Which even avoids canonical momentum build up
  - But with alternating cell polarities, will give more resonances if bending is introduced

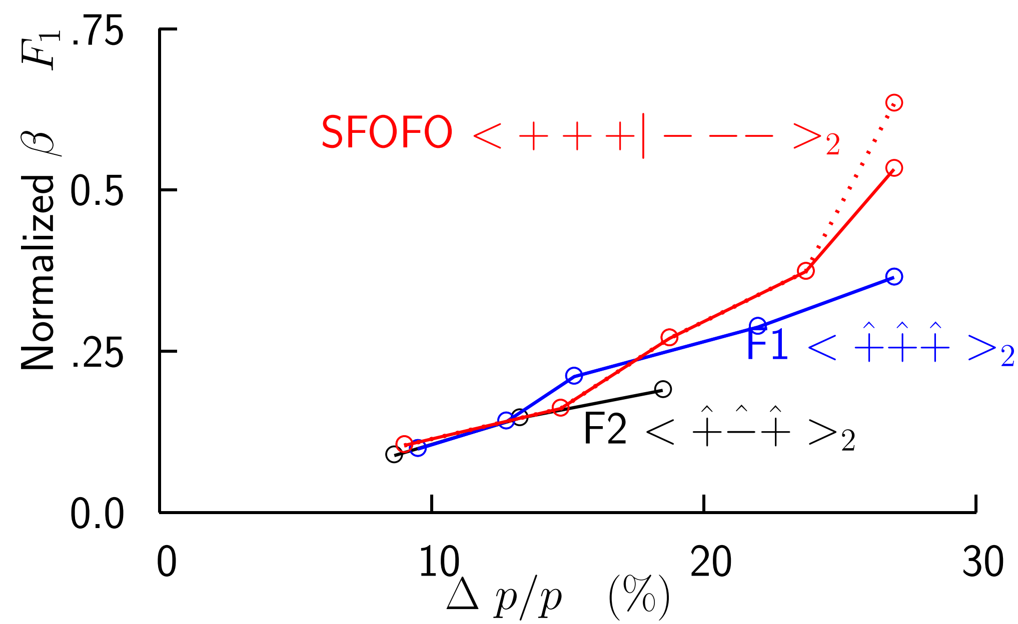




Varying β_*s

- In these cases, the $\Delta p/p$ and βs can be decreased by reducing the coils inside radii and lengths, while adjusting the currents to center the accepted momenta

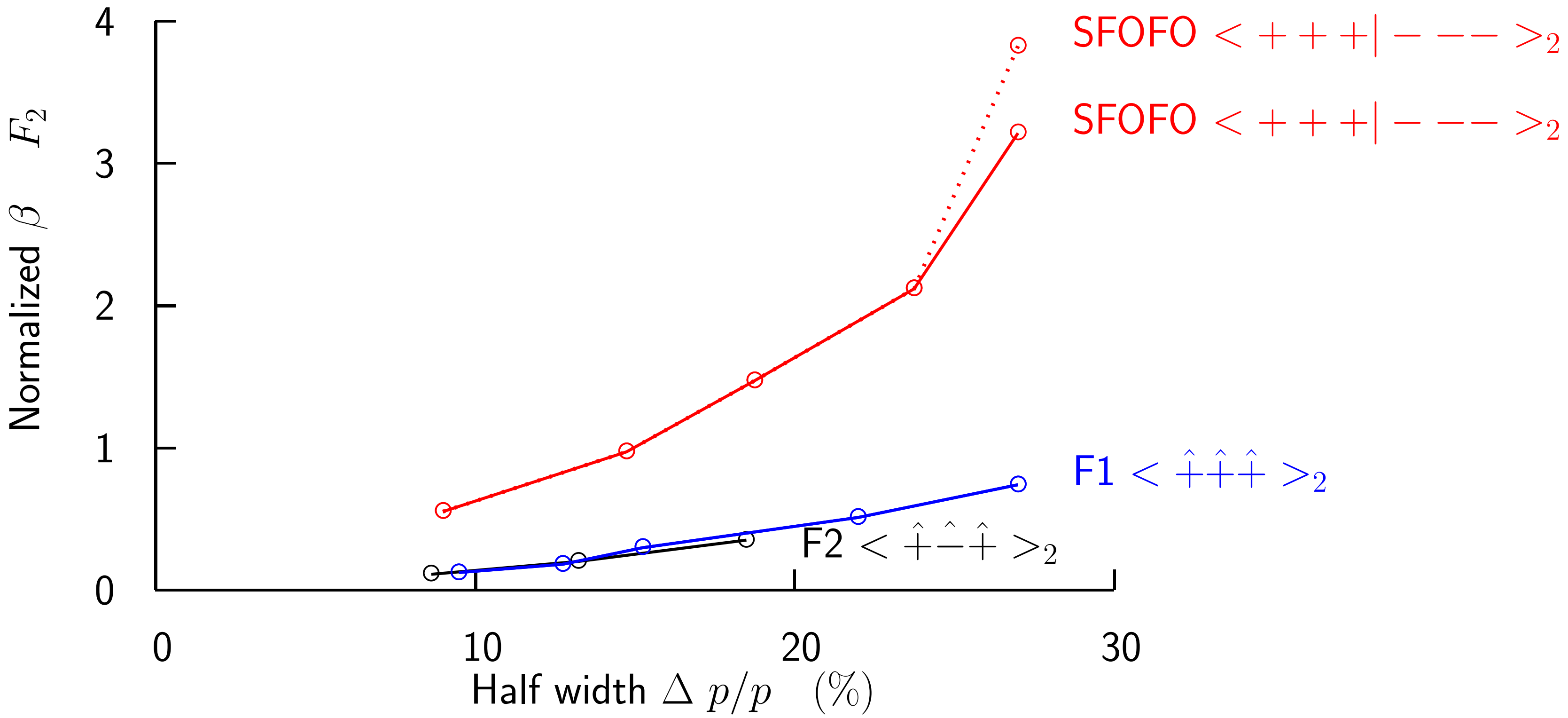




For these new non-flip solutions F1 and F2:

- The F_1s (normalized on the maximum axial fields) are similar to the FS2 SFOFO, and other field flip lattices

Normalized β vs. Current Density: F_2



- Unlike the F_1s, the F_2s of the new lattices are far superior to the flip cases
  These solutions are not dependent on a rapidly changing field at the absorber
  - The A4 lattice has the advantage that canonical momentum will not rise
  - But the A3 lattice may be prefered with bending for emittance exchange
    Separate occasional field reversals may be acceptable in this case

Scaled Lattices for $\Delta p/p \approx 9\%$

If all dimensions cooled equally: $J_x = J_y = J_z = 2/3$

- The equilibrium transverse emittance for LiH absorbers is $0.0061 \times 3/2 \times \beta$
- The equilibrium $\sigma_p/p \approx$ 3 % 3 sigma acceptance is $\Delta p/p \geq 9\%$
- If we use HTS conductor then B is not a limit
- Maximum engineering current densities for HTS cables and tape are approximately 200 A/mm^2

Scaling 4 examples to J=200 A/mm^2:

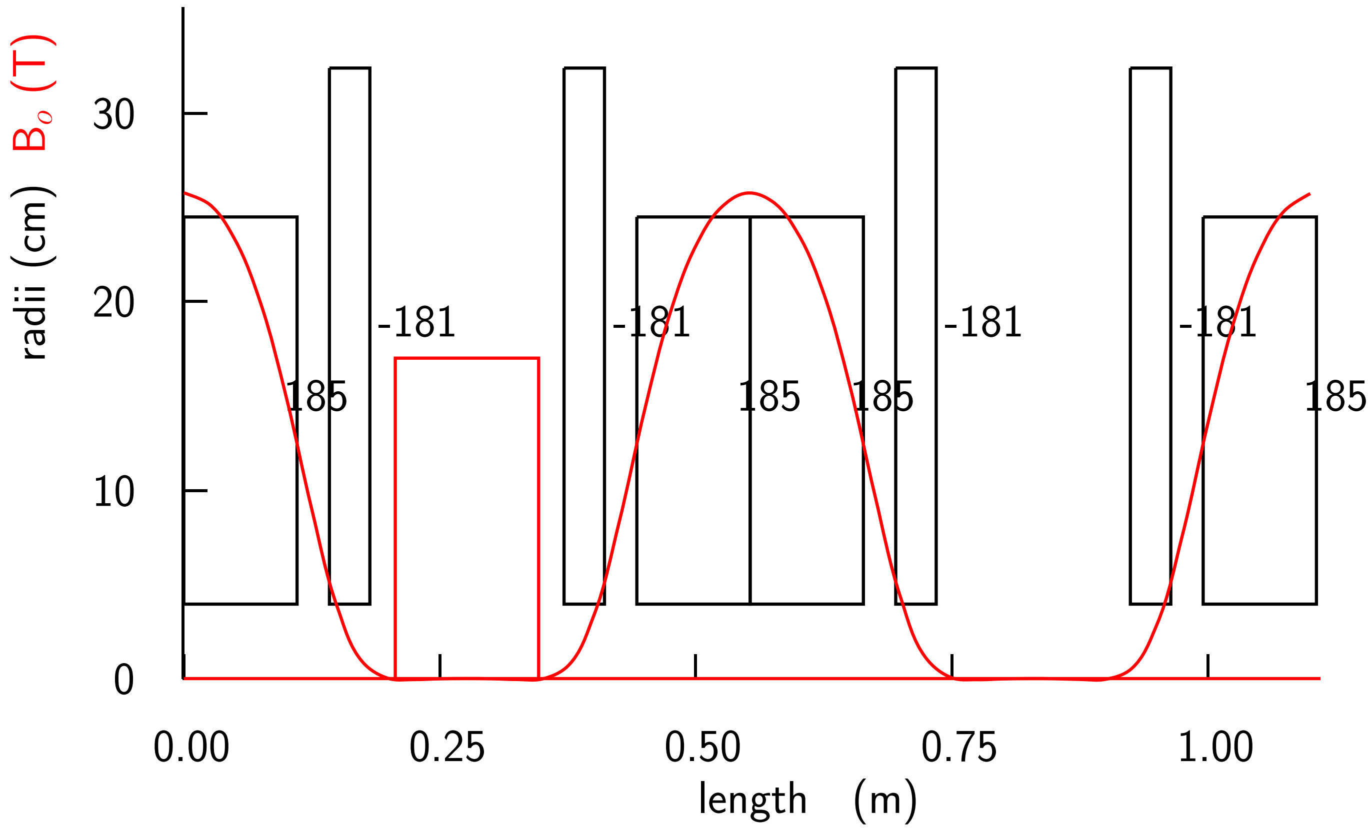
		$\Delta p/p$ %	d m	B_o T	β mm	ϵ_o mm
FS2 SFOFO	$< + + + \| - - - >_2$	9.0	2.7	5.0	55	0.5
RFOFO	$< + - \| + - >_2$	9.25	1.26	7.7	30	0.27
Fernow's F1	$< \hat{+}\hat{-}\hat{+} >_2$	8.75	0.81	21.4	14.8	0.13
Fernow's F2	$< \hat{+}\hat{+}\hat{+} >_2$	9.5	0.79	21.4	12	0.11

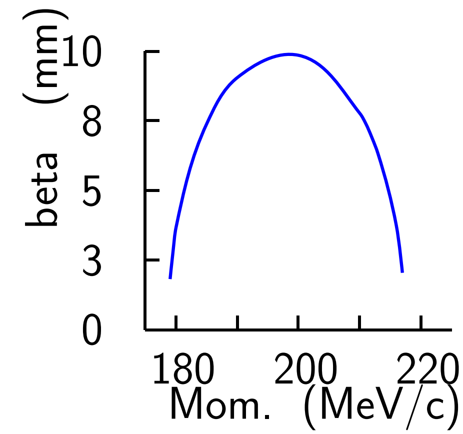
- With Fernow's new lattices, for the same J the βs are down by $\approx$ 3
- This means we need less final solenoid cooling
- And will end with less longitudinal emittance

8) THREE COIL PER CELL, 2ND PASS BAND

Fernow's New Lattices + Bucking Coils $<\hat{+}--\hat{+}--\hat{+}>_2$

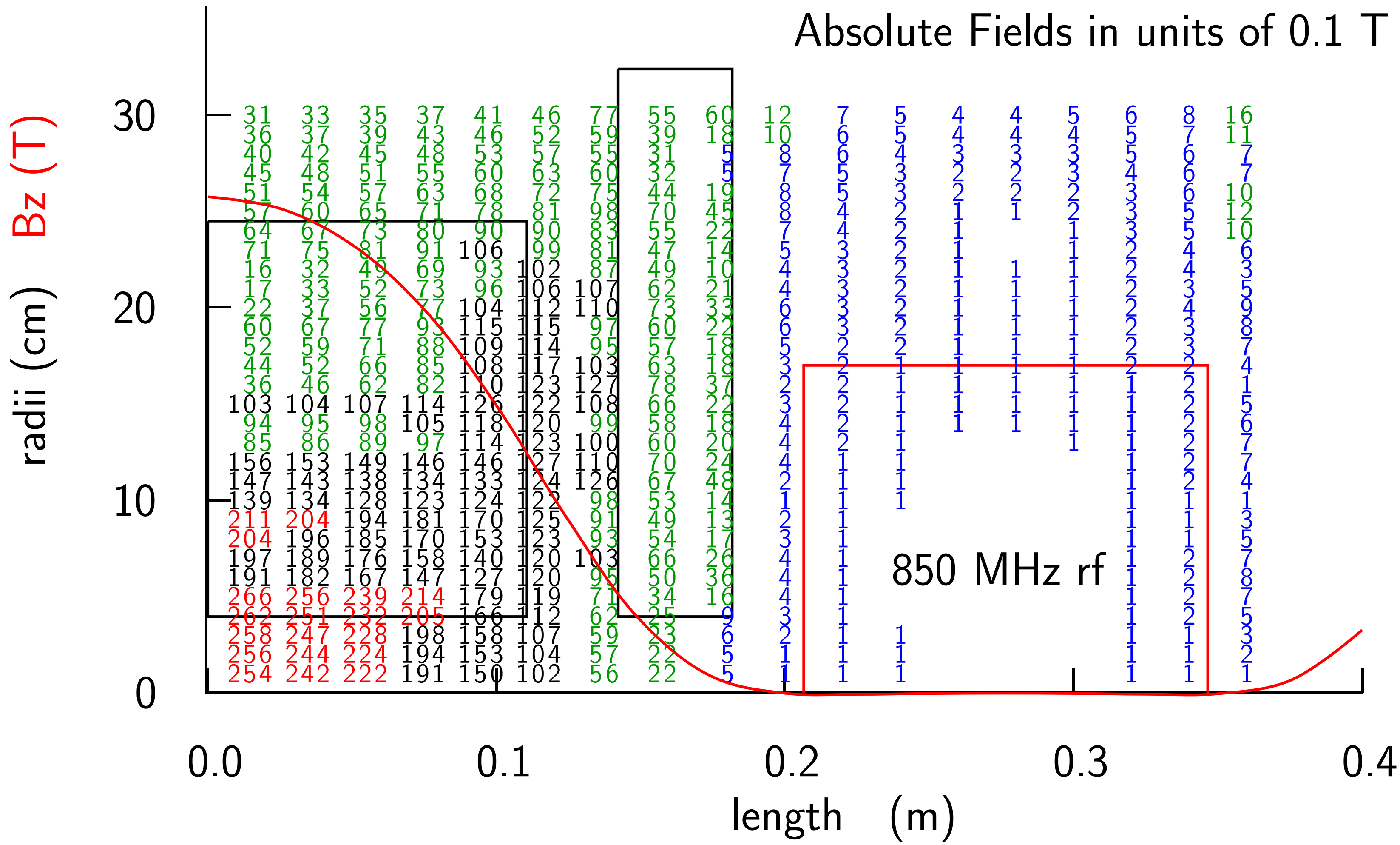
- It may be desirable to add bucking coils to reduce the field on the RF





- The design gave the same 9.6 % $\Delta p/p$ (c.f. 9.5 %)
- With the same current densities ($<$ 200 A/mm^2)
- And, surprisingly, a lower $\beta = 10$ mm (c.f. 12 mm)
- With no field between coils, reversals can be introduced, or left out, without disturbing transverse dynamics

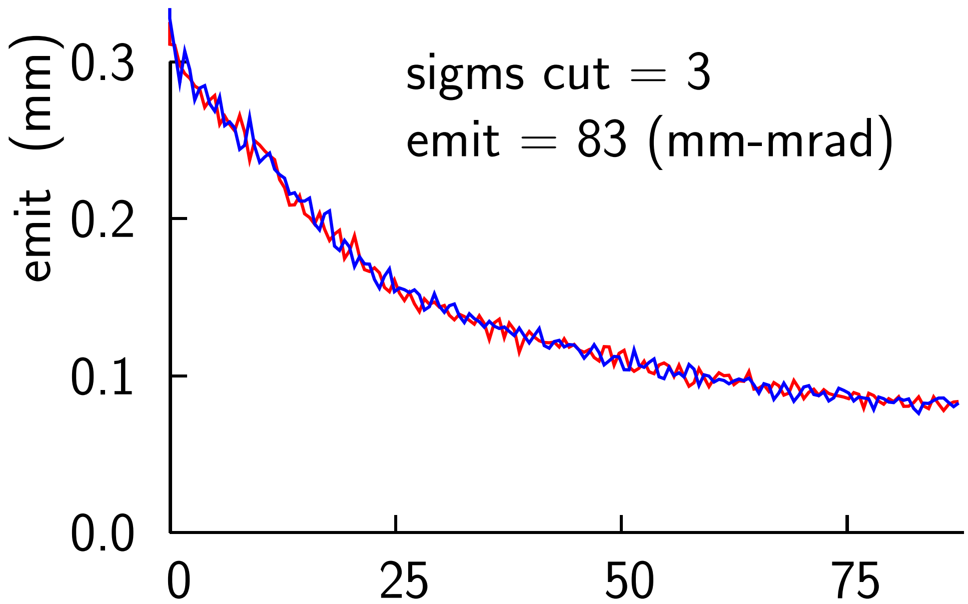
Fields at RF

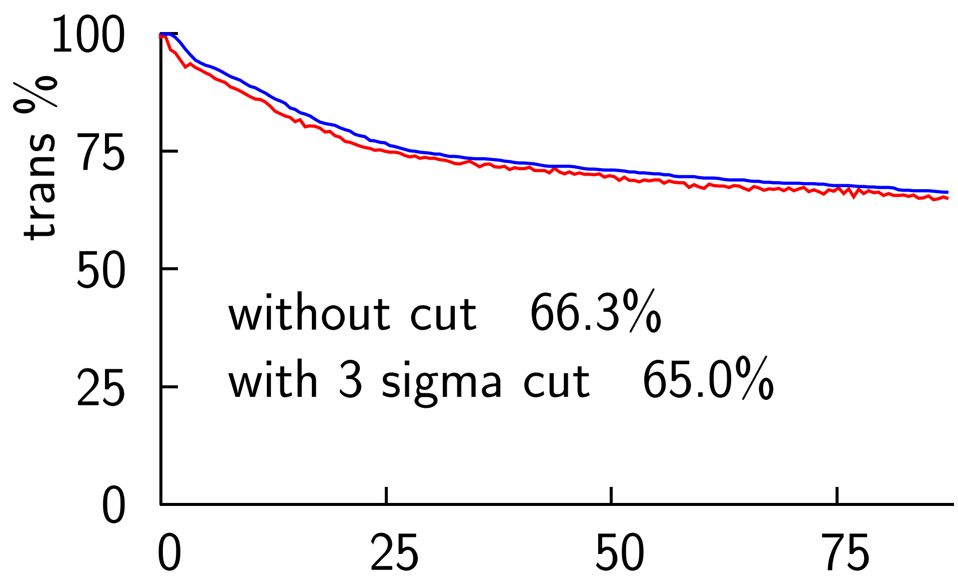


- Field at the rf are less than 0.2 T
- If improved to < 0.1 T, Superconducting cavity could be used

PRELIMINARY: ICOOL simulation of straight channel

- 17 mm long LiH absorber
- 14 cm long 850 MHz rf at 42 MV/m and 41 degrees
- Fano method for scattering

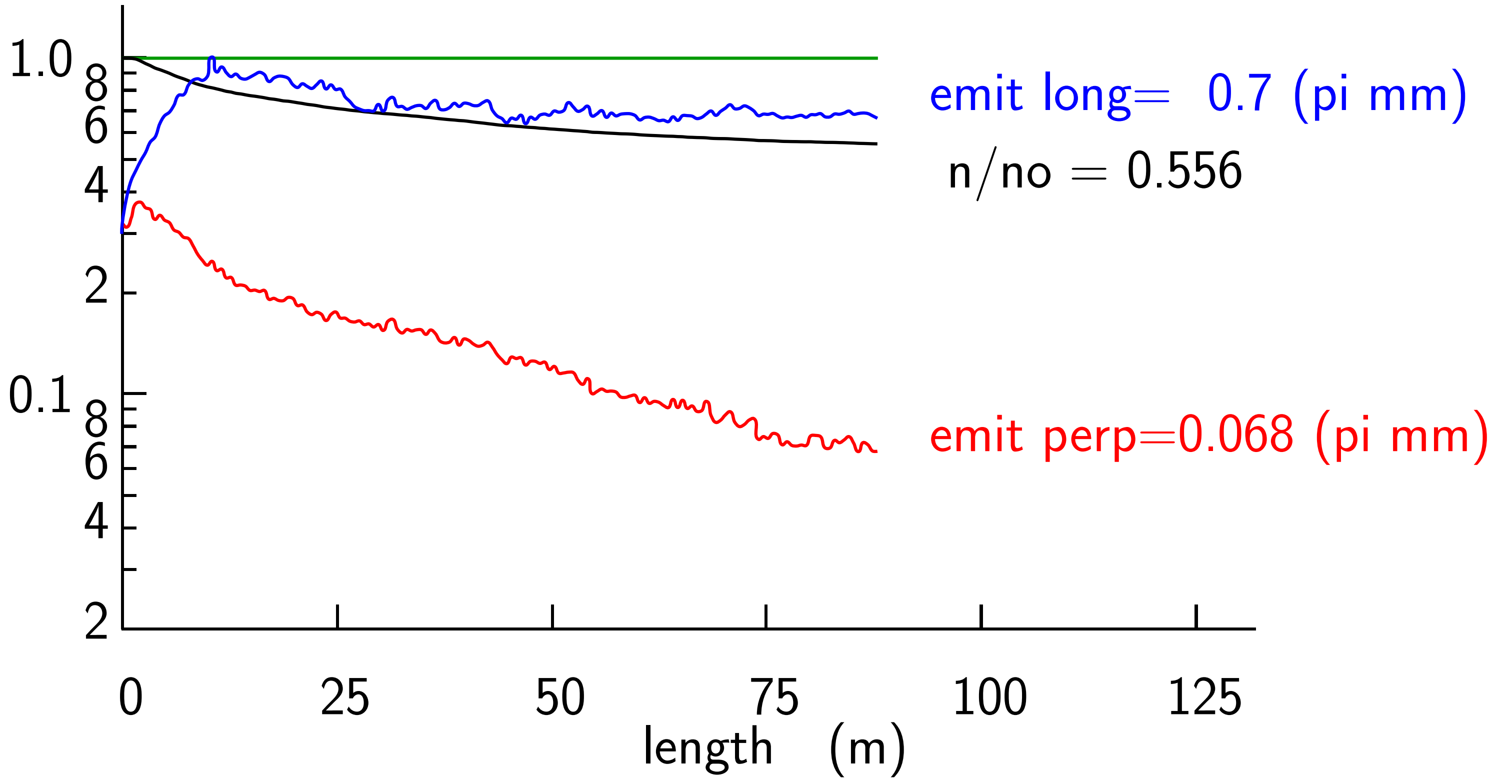




- Cooling to 83 mm-mrad
- But longitudinal emittance rising fast
- We need dispersion and a wedge

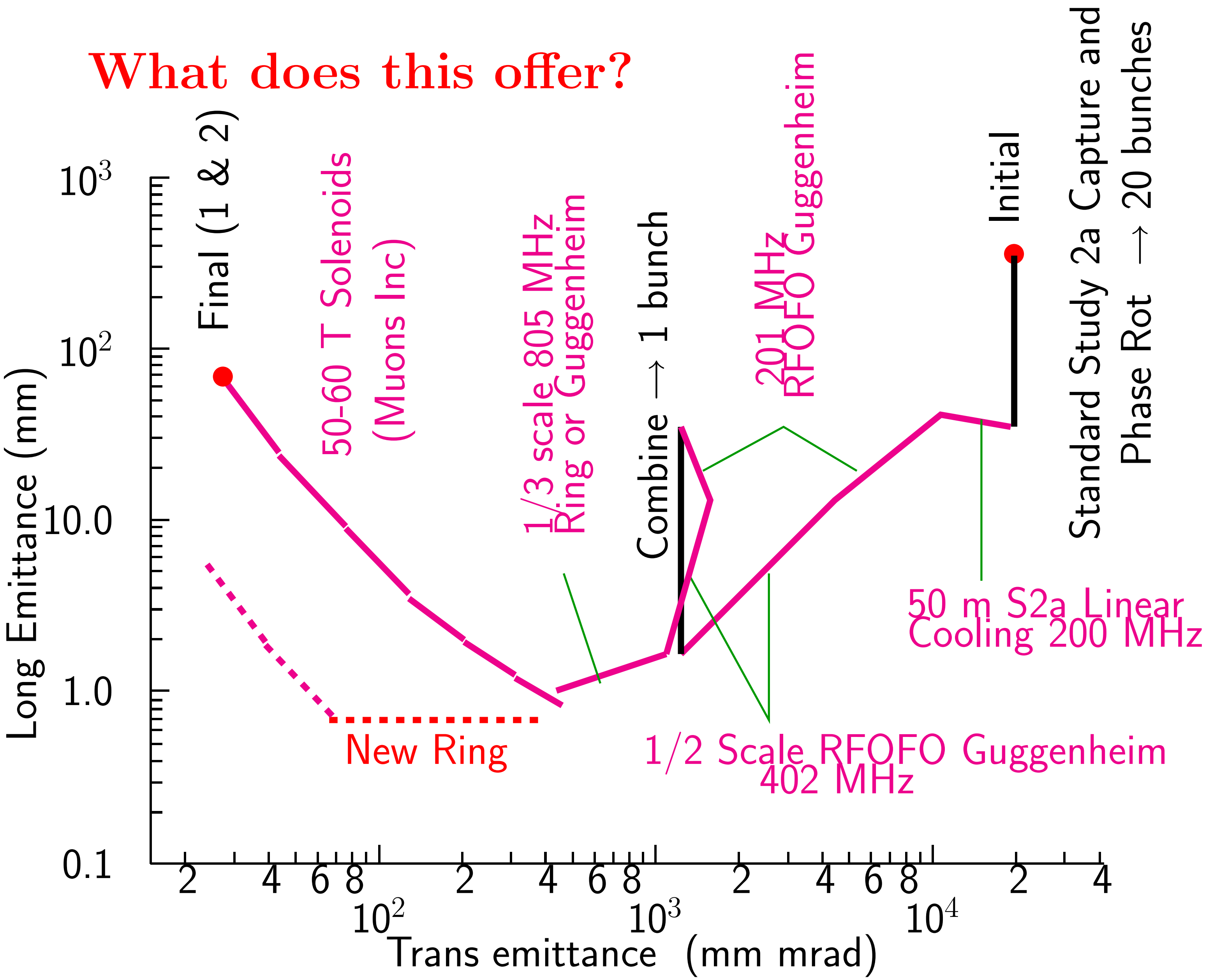
VERY PRELIMINARY: ICOOL simulation of Ring

- 0.125 T vertical field: 33 m circumference
- 90 degree apex LiH wedges
- 14 cm long 850 MHz rf at 42 MV/m and 41 degrees



- Poor input matching
- Cooling to 68 (mm-mrad)
- Longitudinal emittance stable at 2.5%

What does this offer?



- Much lower final longitudinal emittance
- Less momentum spread in Collider Ring
- Or maybe lower transverse emittance

Conclusion

- We have compared the $\Delta p/p$ vs β for fixed B or J, for several lattice
  - Lattices $< +|+ >_1$ or $< +|- >_1$ in the first stop band have the largest $\Delta p/p$
  - 2 coil SFOFO Lattices $< + + | - - >_2$ achieve lower βs for $\Delta p/p \ < \ 22\%$
  - 3 coil SFOFO $< +++|--- >_2$ Lattices are "tunable" to aid "tapering" a channel
  - Lattices without field alternation $< + - | + - >_2$ have fewer resonances when bending is introduced for emittance exchange
  - Lattices without a "field flip" across the absorber require lower current densities and have lower peak fields off the axis, but need field reversals eventually
  - Fernow has introduced simple lattices $< \hat{+}\hat{-}\hat{+} >_2$ with absorber under each coil that offer substantially lower betas for a given coil current density
  - A version of these new lattices $< \hat{+} - -\hat{+} - -\hat{+} >_2$ has near zero field at the RF and looks very attractive for improving the Collider scheme
  - ICOOL simulation of linear channel: $\epsilon_\perp$ ≈80 (mm-mrad) & rising $\epsilon_\parallel$
  - Preliminary ICOOL Simulation of ring: $\epsilon_\perp$ ≈70 (mm-mrad) & stable $\epsilon_\parallel$
- Next Steps
  - Further Study of Ring, eg why is $\epsilon_\perp$ lower in ring than straight
  - Look at magnet

Appendix

1) A1 periodic non-alternating solenoids $< +|+ >_1$

g=70 a=70 ℓ=30 t=14 (cm) J=39 (A/mm^2)

$\pi/2$ Resonance at 100 MeV/c

Operating varied to give different momentum acceptances

2) A2 periodic alternating solenoids $< +|- >_1$

g=70 a=60 ℓ=40 t=14 (cm) J=91 (A/mm^2)

$\pi/2$ Resonance at 100 MeV/c

Operating varied to give different momentum acceptances

3) 2 coil SFOFO $< ++|-- >_2$

For all cases: d=2.75 (m) ℓ=0.5 (m) a=0.3 (m) t=0.11 (m)

	g m	Δp MeV/c	β cm	Bz(max) T	J(max) A/mm^2
2 coil SFOFO	1.0	bad	bad	2.95	36
	0.8	90	61	3.0	37
	0.6	75	31.3	3.1	39
	0.4	63	19.4	3.4	45
	0.2	55	12.7	3.75	53
	0.05	50	9.5	4.1	64

4) 3 coil MICE/FS2 SFOFO $< + + + | - - - >_2$

For all cases: d=275 g=35 a_f=25.5 ℓ_f=16.7 t_f=9 a_c=69 ℓ_c=30 t_c=10 (cm)

	J_f A/mm^2	J_c A/mm^2	Δp MeV/c	β cm	Bz(max) T
3 coil SFOFO	117	89	108	42(50)	3.4
	139	79	75	25.4	3.9
	155	67	75	16.7	4.3
	171	49	59	10.5	4.1
	193	0	36	5.6	4.9

5) RFOFO with a=77 cm $< + - | + - >_2$

Radius chosen to go over the 200 MHz cavities

d=275 a=77 ℓ=50 t=11 (cm)

RFOFO	g m	a m	Δp MeV/c	β cm	Bz(max) T	J A/mm^2
RFOFO rings	.7	.77	87	42.7	2.75	95
shifted coils	.3	.77	80	35	2.81	105

6) RFOFO with smaller radii $< + - | + - >_2$

For all cases: d=275 ℓ=50 t=11 (cm)

RFOFO	g	a	Δp	β	Bz(max)	J
	m	m	MeV/c	cm	T	A/mm^2
	.7	.35	82	40	3.0	43
	.3	.35	59	16	3.5	54
	.1	.35	66	11.6	3.8	66
d	0	.35	49	10.6	4.0	77
f	0	.15	37	4.6	4.85	80

7) Fernow's A3 & A4 $< \hat{+}\hat{+}\hat{+} >_2$ and $< \hat{+}\hat{-}\hat{+} >_2$

All cases: d=2.0 m p=200 MeV/c

	a	L	t	Δp	β	B_z(max)	J
	m	m	m	MeV/c	cm	T	A/mm^2
A3	.7	.74	.42	108	27	3.6	15
	.5	.74	.42	88	18	4.1	15
	.3	.74	.42	61	10.4	5.4	16
	.2	.5	.42	51	5.2	7.2	24
	.14	.5	.32	38	3.1	8.45	31
A4	.7	.74	.52	77	9.9	5.1	25
	.3	.74	.42	53	6.5	6.0	19
	.2	.5	.42	40	3.9	7.7	27
	.14	.5	.32	35	2.7	8.7	33

8) Fernow's A3 with Bucking Coils $< \hat{+} - - \hat{+} - - \hat{+} >_2$

p MeV/c	d cm	a_f cm	ℓ_f cm	t_f cm	J_f A/mm^2	g_b cm	a_b cm	ℓ_b cm	t_b cm	J_b A/mm^2	Δp MeV/c	β cm	B_z(max) T
200	79	4.0	19.7	20.5	189	5.5	6.3	4	20.5	196	38	0.99	26.1