Helical 200MHz FOFO snake (vacuum RF)

1. The follow-up of the 09/11/08 discussion of a planar FOFO snake (solenoids tilted and/or displaced in one plane only, e.g. vertical)
2. First results with helical FOFO snake – also suitable for cooling of both μ+ and μ-!

The next 3 slides are from 09/11/08 presentation to remind the problems encountered with a planar FOFO snake .

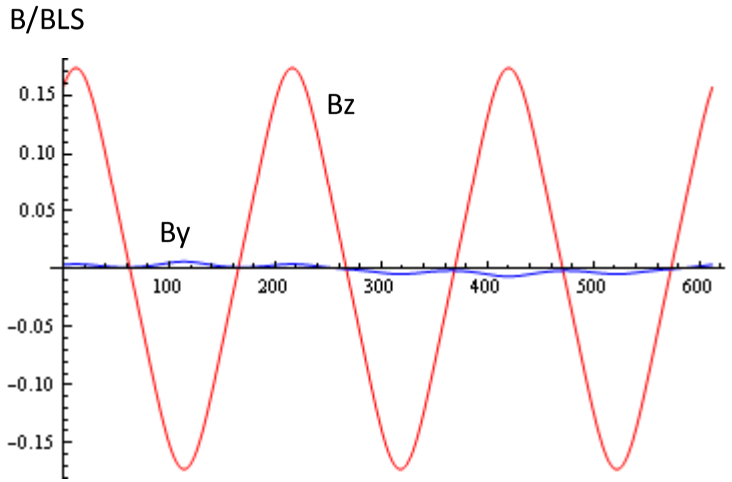
New channel parameters:

200 MHz pillbox RF 2x36cm, Emax=16MV/m

Solenoids: L=24cm, Rin=40cm, Rout=72cm,
vertical pitch (mrad): +20, -30, +20, -20, +30, -20, no vertical offset

Absorbers: LH2, total width (on-axis) 6x15cm, horizontal wedge angle tan(α/2)=0.05

Total length of 6-cell period 6.12m



Bz_max=2.6T (BLS=15)
requires j=37.3A/mm^2

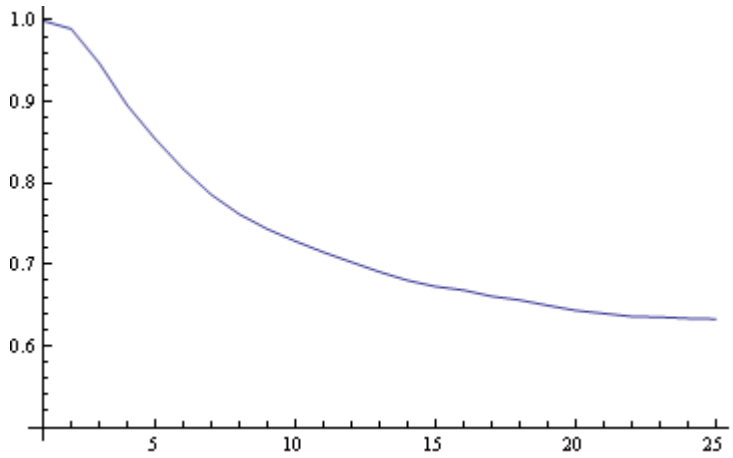
mode	I	II	III
tune	1.378 + 0.005 I	1.308 + 0.014 I	0.167 + 0.002 I
ε_eq (mm)	6.8	2.6	5.9

Tracking

- Lie-transform from "true" invariants $J_{I, II, III}$ to phase space variables used transverse amplitude-energy correlation taken into account
- 1330 particles distributed in tetrahedron

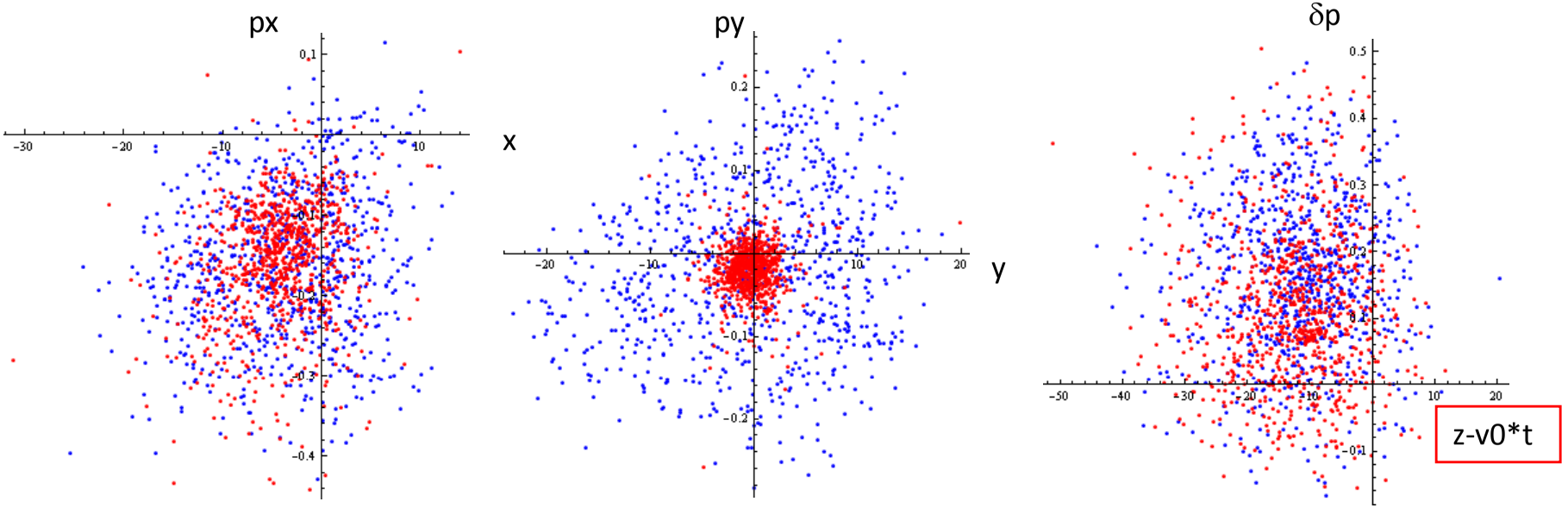
  $(J_I + J_{II})/2.34 + J_{III}/3.6 < 1$ (cm)
- Phases chosen at random
- No stochastic force so far

Survival after 25 periods (153m) 63%



Evolution of surviving particles distribution

blue – initial position, red – after 25 periods



Emittances (cm)	initial	final
6D	7.95	0.23
Trans. average	1.92	0.39
Longitudinal	4.37	3.78

$$\delta_p = \frac{\gamma - \gamma_0}{\beta_0^2 \gamma_0},$$

$$\frac{\beta\gamma}{\beta_0\gamma_0} = 1 + \delta_p - \frac{\delta_p^2}{2\gamma_0^2} + \ldots$$

Problems with a planar FOFO snake:

1. Large imbalance in the cooling rates of the transverse modes – all longitudinal cooling at the expense of one mode, the other gets parasitically stronger.
2. Significantly smaller transverse acceptance as compared to a straight FOFO channel (without tilts) – partly due to "undercooling" of one of the modes, partly due to nonlinear field components.

Valeri Balbekov suggested to increase coil radii to reduce nonlinearities. It worked: an increase by 20 cm improved transmission over 25 periods (153m) 63% $\rightarrow$ 73%

To equalize the cooling rates of the transverse modes Bob Palmer suggested to add a constant component to alternating solenoidal field. It worked surprisingly well!

J-/J+	Q1	Q2	Q3	transmission
1	1.3954+0.0050i	1.3294+0.0149i	0.1705+0.0015i	0.73
0.990	1.3970+0.0067i	1.3197+0.0128i	0.1675+0.0018i	0.64
0.978	1.4113+0.0106i	1.3046+0.0083i	0.1603+0.0024i	0.38

- Unfortunately, strong coupling introduced by the ~ constant Bz component destroys transverse acceptance.

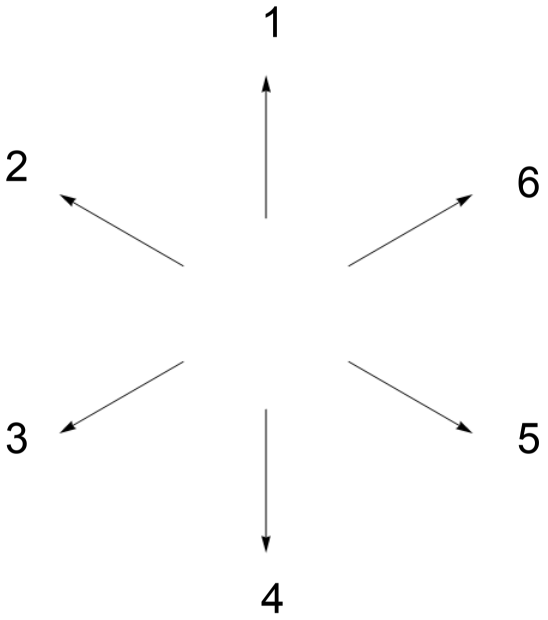
Helical FOFO snake – the idea:

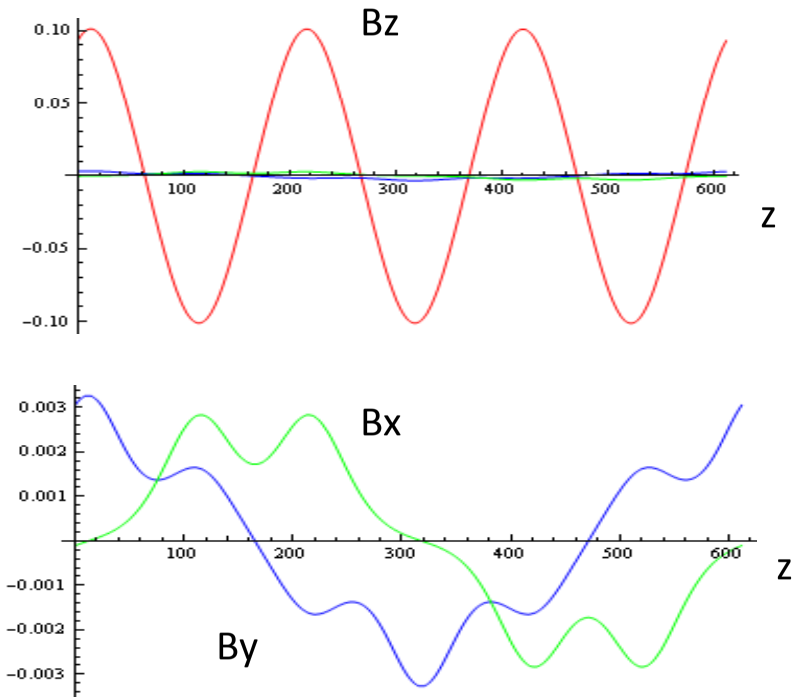
Create rotating $B_\perp$ field by tilting (or displacing) solenoids in planes $x*\cos(\phi_k)+y*\sin(\phi_k)=0$, k=1,2,…

Example for 6-cell period:

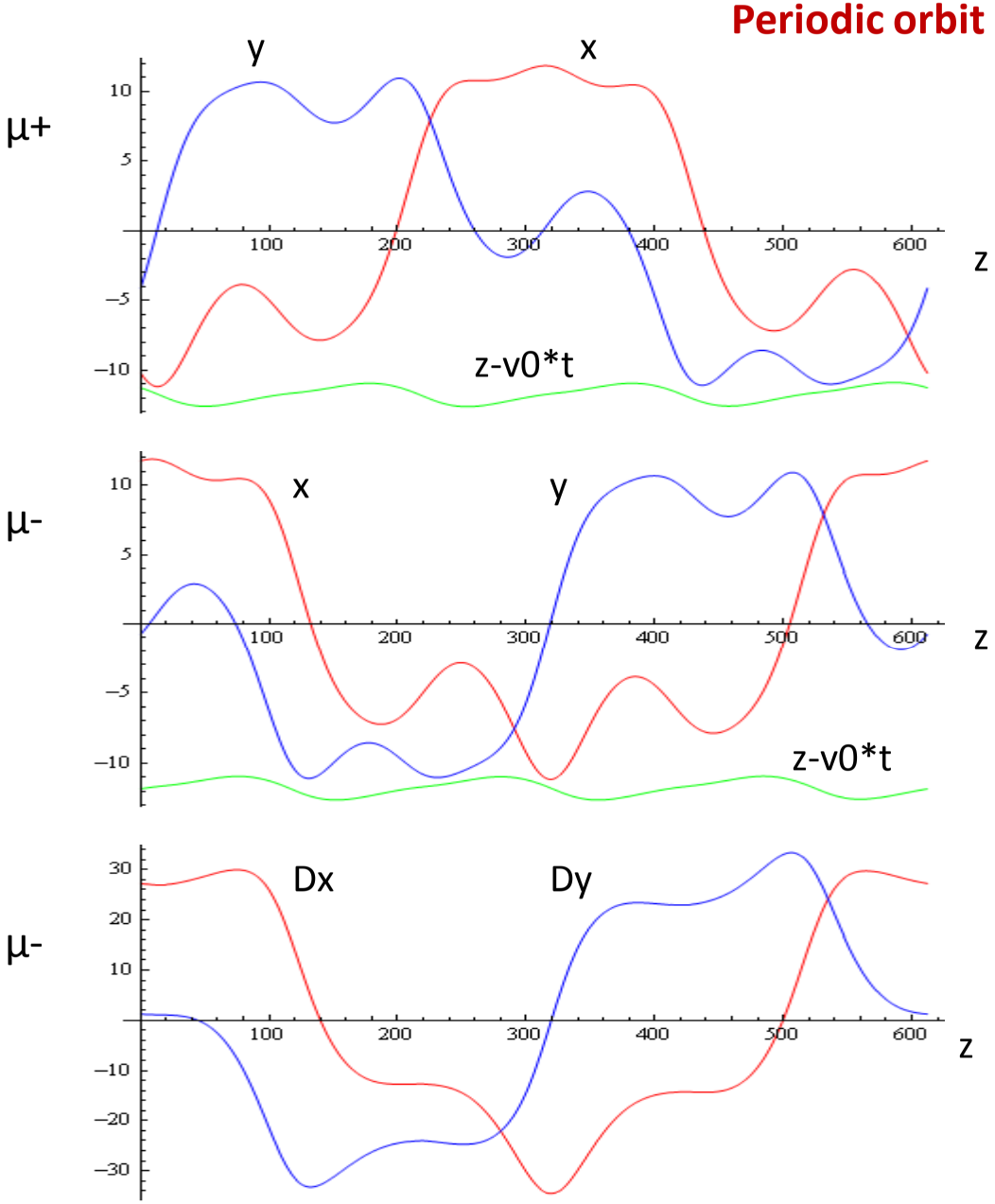
Solenoid #	1	2	3	4	5	6
Polarity	+	-	+	-	+	-
Roll angle ϕ_k	0	$2\pi/3$	$4\pi/3$	0	$2\pi/3$	$4\pi/3$

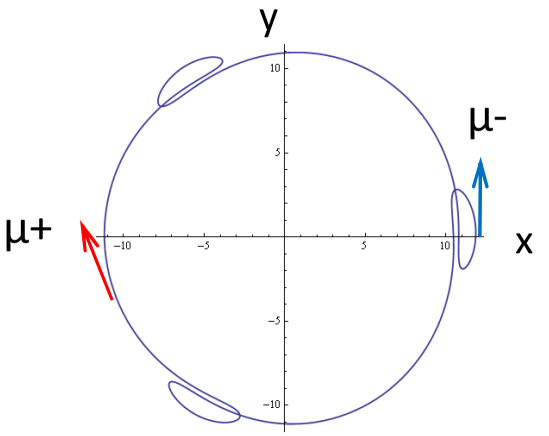
With 20mrad pitch angle:





Periodic orbit





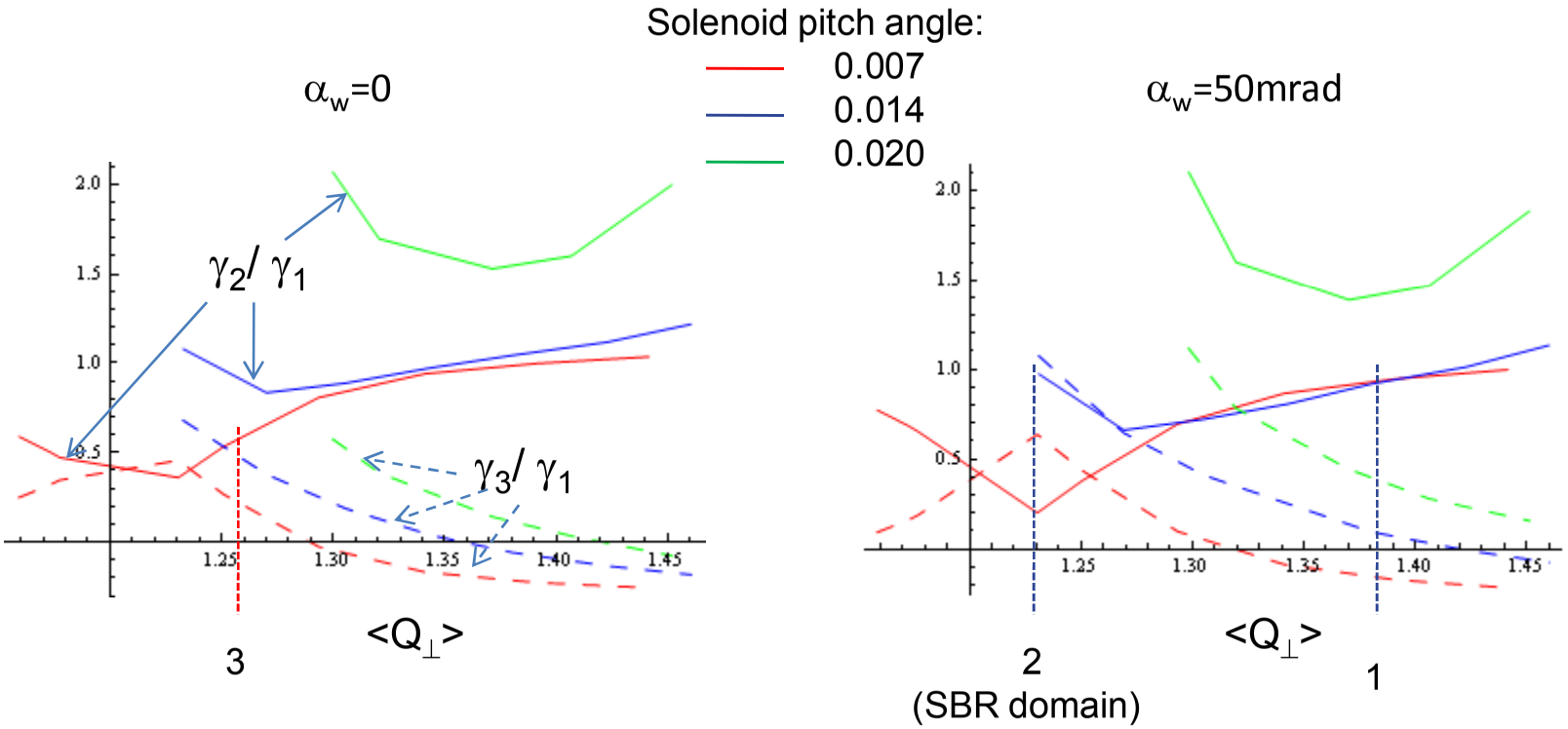
Helical FOFO snake – good for cooling both μ+ and μ-!

For the particular example with Bzmax=2.5T and wedge half-angle α_w=50mrad :

mode	I	II	III
tune	1.3236+0.0075i	1.4169+ 0.0105i	0.1371+ 0.0033i
ε_eq (mm)	4.7	3.4	3.7

overshoot!

Cooling rates ratios vs channel parameters:



Tracking studies for 3 cases:

pitch	wedge	Q1	Q2	Q3	trans
0.014	0.07	1.367+0.011i	1.406+0.009i	0.179+0.002i	0.77
0.014	0.04	1.158+0.007i	1.305+0.007i	0.065+0.007i	0.15
0.007	0	1.239+0.012i	1.279+0.007i	0.181+0.002i	0.98

- good transmission in the 3rd case was partly due to a smaller nonlinear distortion of the initial distributions of particles in the phase space (it was more compact), partly due to weaker detuning and resonances.

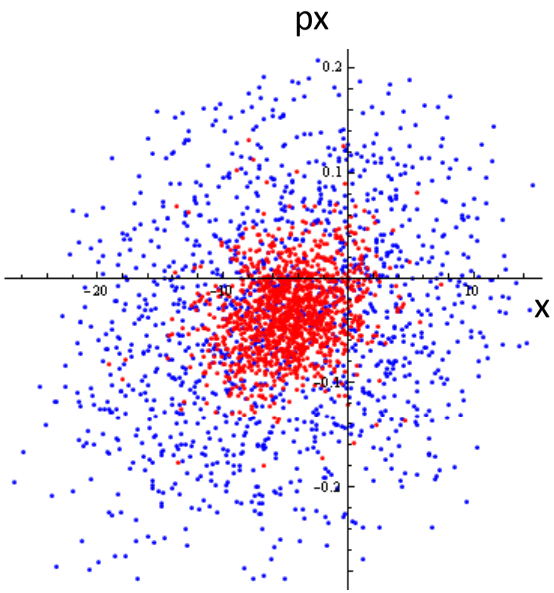
Perturbation theory (3rd case):

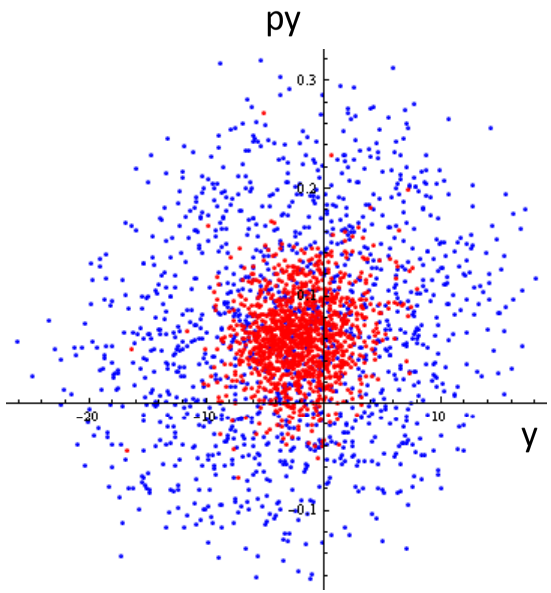
	∂Q1/	∂Q2/	∂Q3/
/ ∂J1	{-0.0168499 + 0.000444829 i,	-0.0159542 - 0.00315035 i,	-0.0126387 + 0.00197689 i}
/ ∂J2	{-0.0167346 - 0.00268546 i,	-0.0117902 + 0.000979594 i,	-0.0194624 + 0.000825513 i}
/ ∂J3	{-0.0130235 - 0.000957154 i,	-0.0193821 - 0.00254194 i,	-0.0067989 + 0.00174956 i}

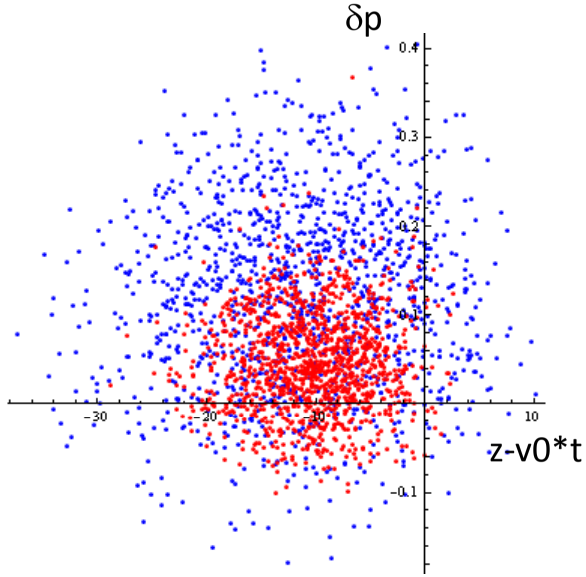
Ji is action variable for i-th normal mode in cm, not multiplied by $\beta\gamma$.
According to this analytical calculations, cooling of the 2nd mode may fail at J1=2.3cm or J3=2.8cm

Evolution of the surviving particles distribution

blue – initial position, red – after 25 periods







Emittances (cm)	initial	final
6D	7.66	0.04
Trans. average	1.81	0.25
Longitudinal	3.47	1.36

Again, no stochastic process in simulations!

Y.Alexahin, MCTF meeting 10/02/2008

Conclusions

- Helical snake allows to equalize transverse cooling rates w/o compromising acceptance
- Just as a planar snake, it works for both μ+ and μ- at the same time
- Probably, the longitudinal cooling rate can be increased as the beam is cooled transversely by gradually increasing the solenoid pitch angle and the absorber wedge angle
- Tracking with initial distribution from Dave's capture/rotation section is underway