Muon Collider Cross Sections

For $\sqrt{s}$ < 500 GeV lepton collider

- threshold regions:
  - top pairs
  - electroweak boson pairs
  - Zh production
- s-channel Higgs production:
  (requires muon collider)
- coupling ∝ mass production

$$\left[\frac{m_\mu}{m_e}\right]^2 = 4.28 \times 10^{\ 4}$$

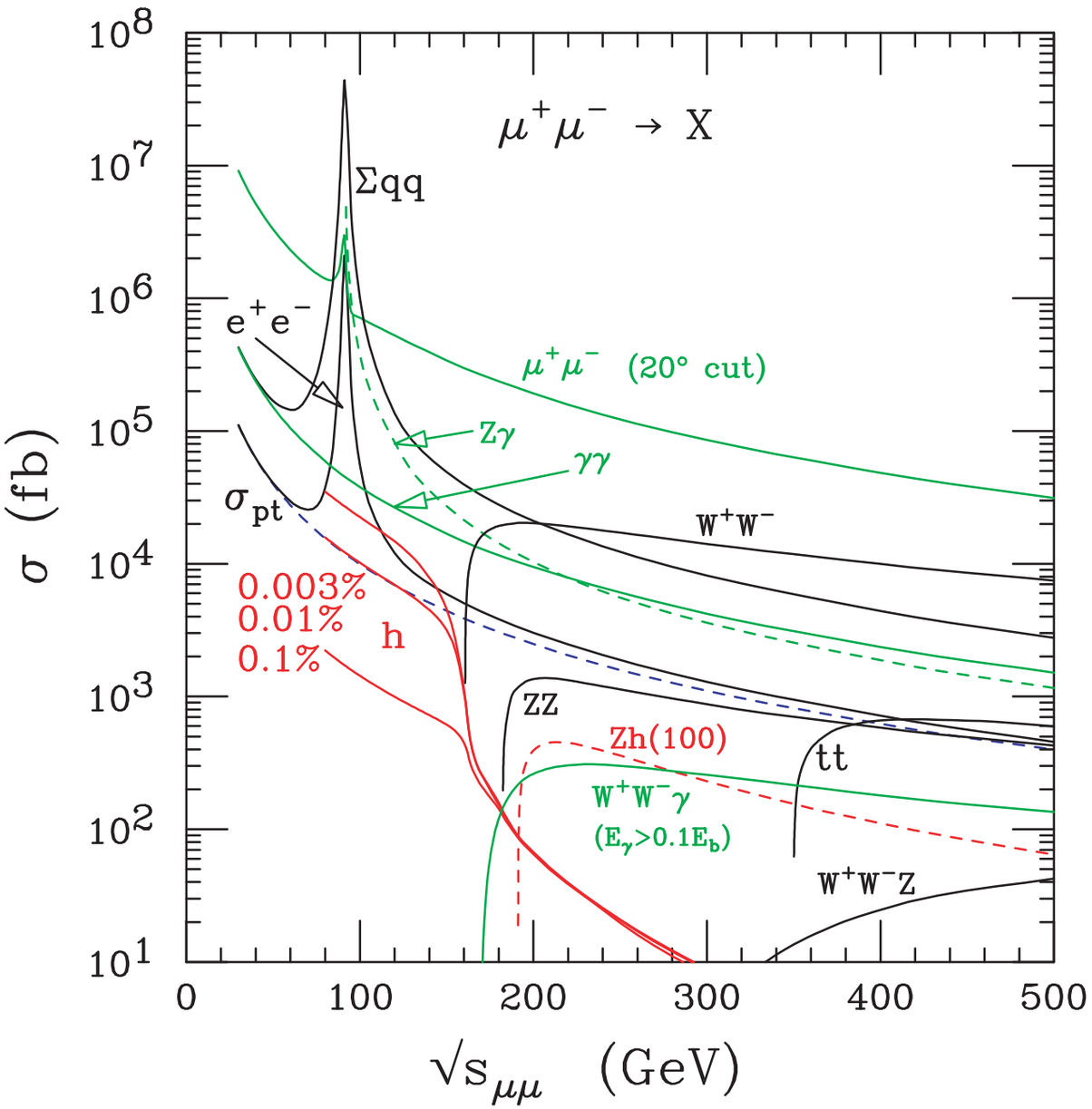
- narrow state

$$m(h) = 110\ \text{GeV}: \quad \Gamma = 2.8\ \text{MeV}$$
$$m(h) = 120\ \text{GeV}: \quad \Gamma = 3.6\ \text{MeV}$$
$$m(h) = 130\ \text{GeV}: \quad \Gamma = 5.0\ \text{MeV}$$
$$m(h) = 140\ \text{GeV}: \quad \Gamma = 8.1\ \text{MeV}$$
$$m(h) = 150\ \text{GeV}: \quad \Gamma = 17\ \text{MeV}$$
$$m(h) = 160\ \text{GeV}: \quad \Gamma = 72\ \text{MeV}$$

- direct width measurement

$$\Delta E/E \approx 0.003\% \ \text{ and more than } 2\ pb^{-1}$$

Standard Model
Cross Sections



E. Eichten, "The Basics of Muon Collider Physics"
(Fermilab-Pub-09-225-T)

For √s > 500 GeV

- Above SM thresholds:
- R essentially flat:

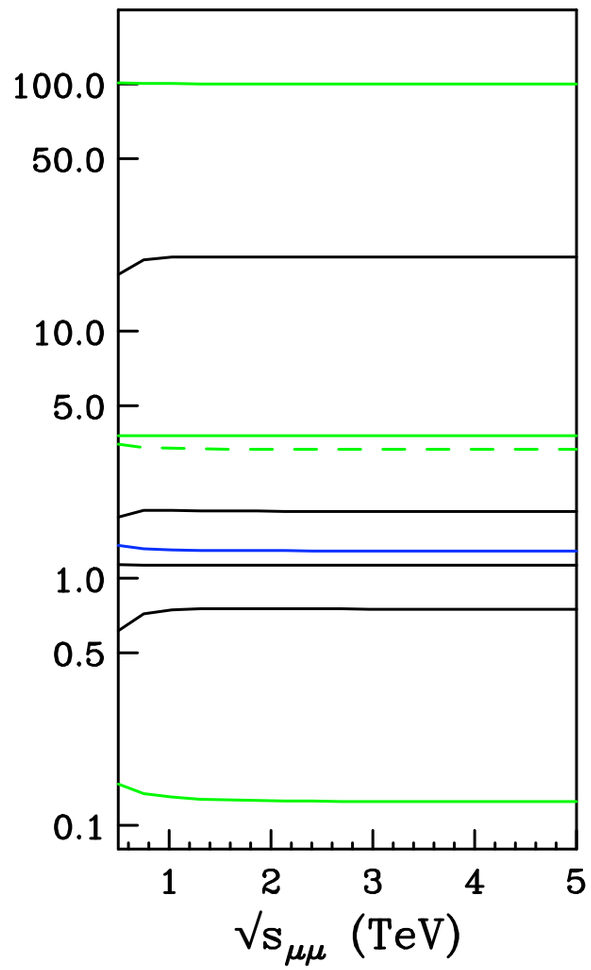
(one unit of R)

$$\sigma_{\rm QED}(\mu^+\mu^- \to e^+e^-) = \frac{4\pi\alpha^2}{3s} = \frac{86.8 \text{ fb}}{s(\text{TeV}^2)}$$

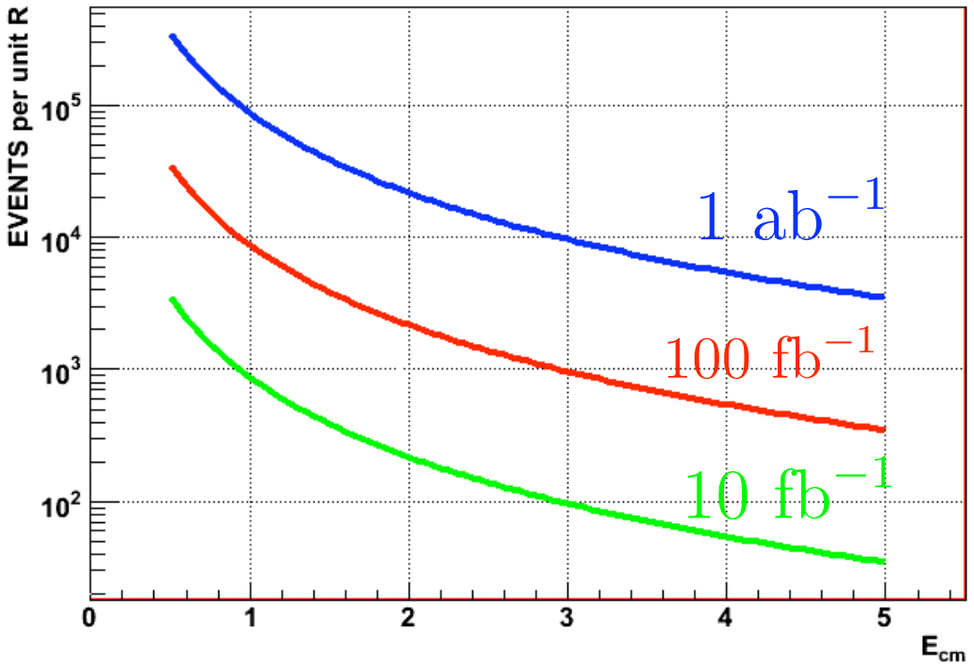
R at √s = 3 TeV

$O(\alpha_{em}^2)$ $O(\alpha_s^0)$

$$\begin{aligned}
\mu^+\mu^-(20^\circ \text{ cut}) &= 100 \\
W^+W^- &= 19.8 \\
\gamma\gamma &= 3.77 \\
Z\gamma &= 3.32 \\
t\bar{t} &= 1.86 \\
b\bar{b} &= 1.28 \\
e^+e^- &= 1.13 \\
ZZ &= 0.75 \\
Zh(120) &= 0.124
\end{aligned}$$



Luminosity Requirements



For example: $\sqrt{s} = 1.5$ TeV ⇒ 3860 events/unit of R

$\mathcal{L} = 10^{34}\ \text{cm}^{-2}\text{sec}^{-1}$

$\rightarrow 100\ \text{fb}^{-1}\text{year}^{-1}$

Total - 510 K SM events per year

Processes with R ≥ 0.01 can be studied

Minimum Luminosity for Muon Collider

Narrow resonances in lepton colliders play a vital role in precision studies

State	$\mathrm{BR}(\mu^+\mu^-)$	Γ/M	
$\phi(1.019)$	2.9×10^{-4}	3.98×10^{-3}	Kaons CPV
$J/\psi(3.097)$	5.9×10^{-2}	3.02×10^{-5}	1D - $D^{\pm,0}$ 3S - D, D*; 2D - D_s
$\Upsilon(9.460)$	2.5×10^{-2}	5.71×10^{-6}	4S - B factory, tau, charm
$Z^0(91.19)$	3.4×10^{-2}	2.74×10^{-2}	precision tests - SM
if $h^0(115)$	2.5×10^{-4}	2.78×10^{-5}	Higgs couplings - EW

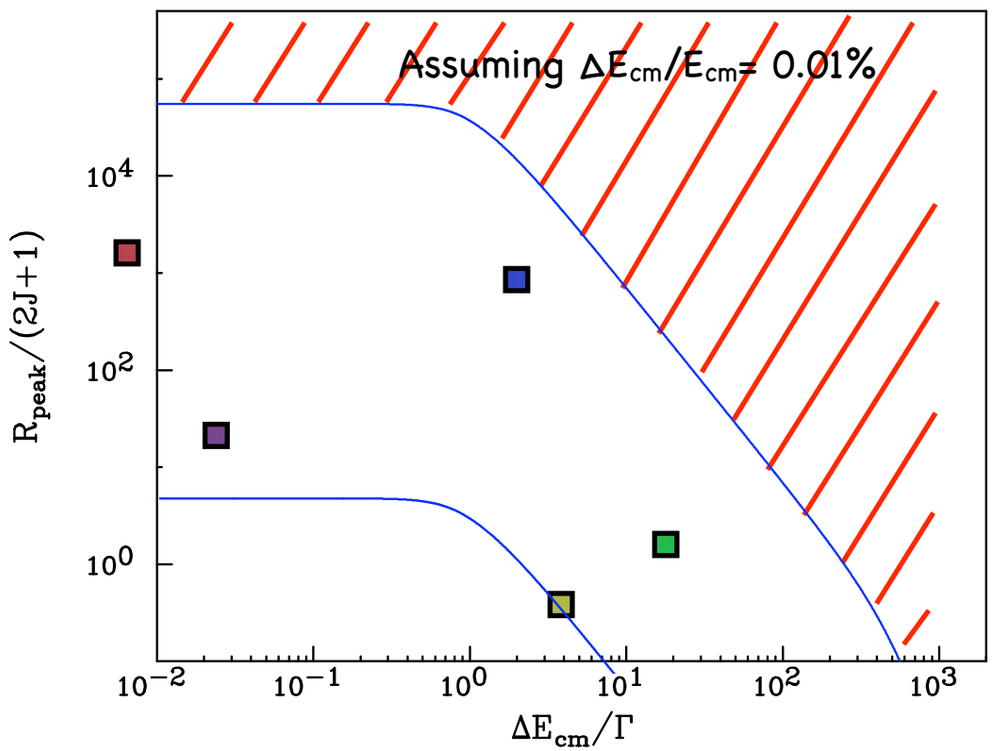
Universal behavior

$$\sigma(E) = \frac{2J+1}{(2S_1+1)(2S_2+1)} \frac{4\pi}{k^2} \left[\frac{\Gamma^2/4}{(E-E_0)^2 + \Gamma^2/4} \right] B_{in} B_{out}$$

Convolute with beam spread

$$\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(E-E_0)^2}{2\sigma^2}\right)$$

$$\rightarrow \Delta E_{\mathrm{cm}}/E_{\mathrm{cm}} = 2\ln(2)\sigma$$



Can use to set minimum required luminosity

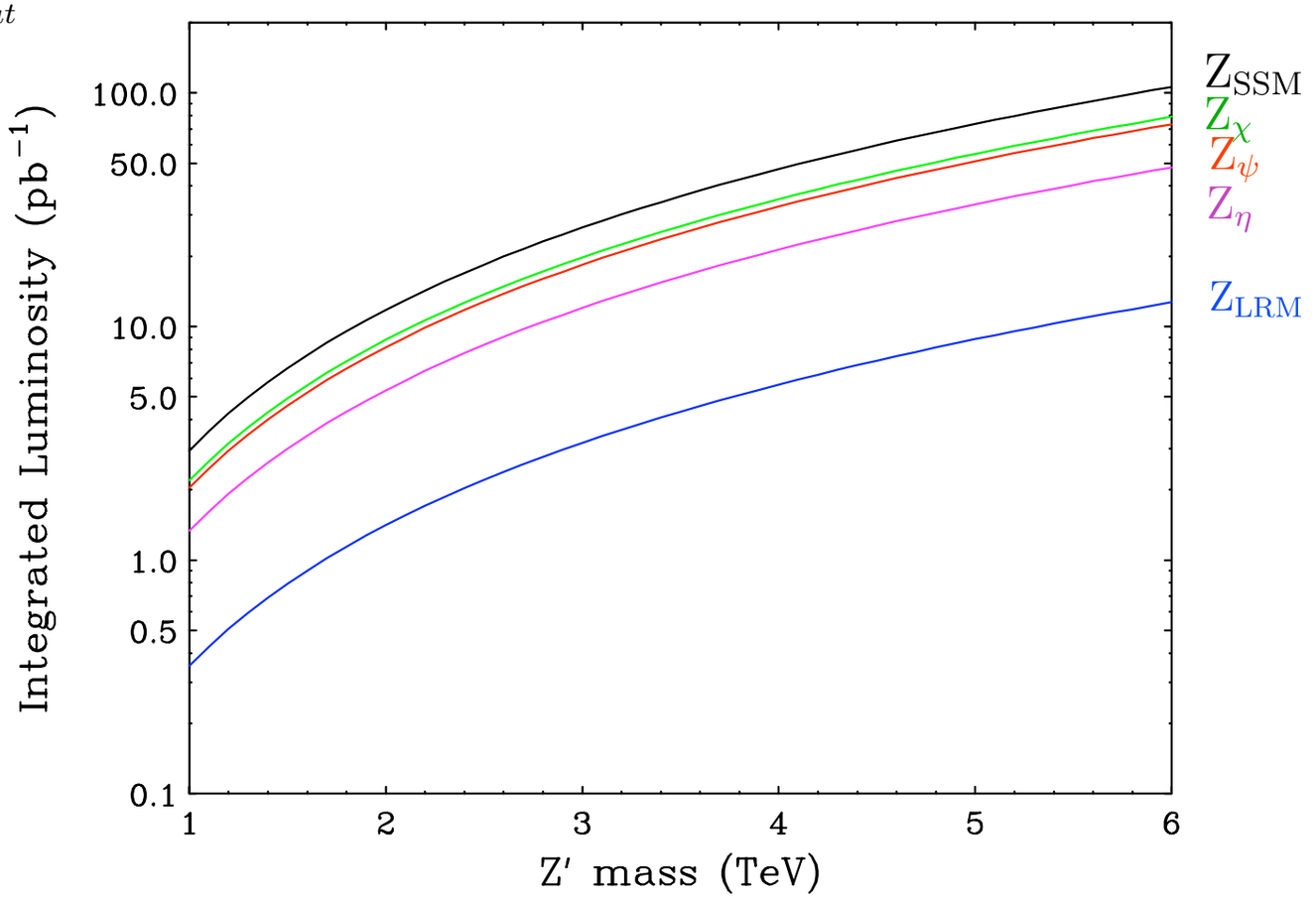
- For a narrow resonance with $2\Delta E_{beam} / \Gamma_{resonance} << 1$:

$$\sigma(E) = \frac{2J+1}{(2S_1+1)(2S_2+1)} \frac{4\pi}{k^2} \left[\frac{\Gamma^2/4}{(E-E_0)^2 + \Gamma^2/4} \right] B_{in} B_{out}$$

$$\rightarrow R_{\rm peak} = (2J+1) 3 \frac{B(\mu^+\mu^-) B(visible)}{\alpha^2_{\rm EM}}$$

- For new gauge boson: Z′
  - examples: SSM, E6, LRM
  - 5σ discovery limits: 4-5 TeV at LHC (@ 300 fb^{-1})

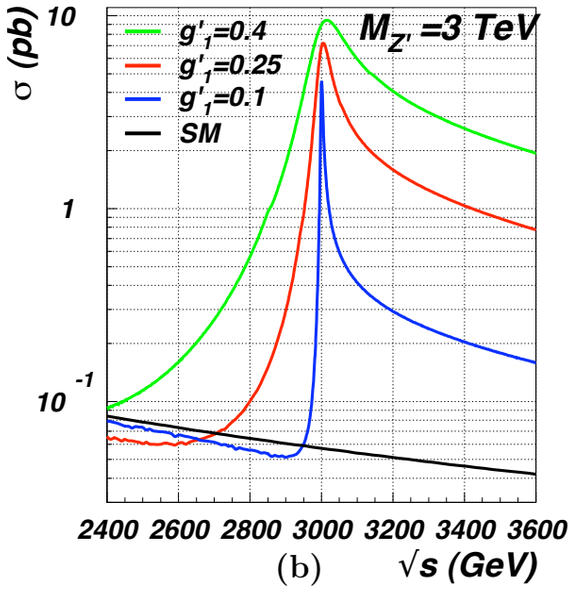
The integrated luminosity required to produce 1000 $\mu^+\mu^-$ -> Z′ events on the peak

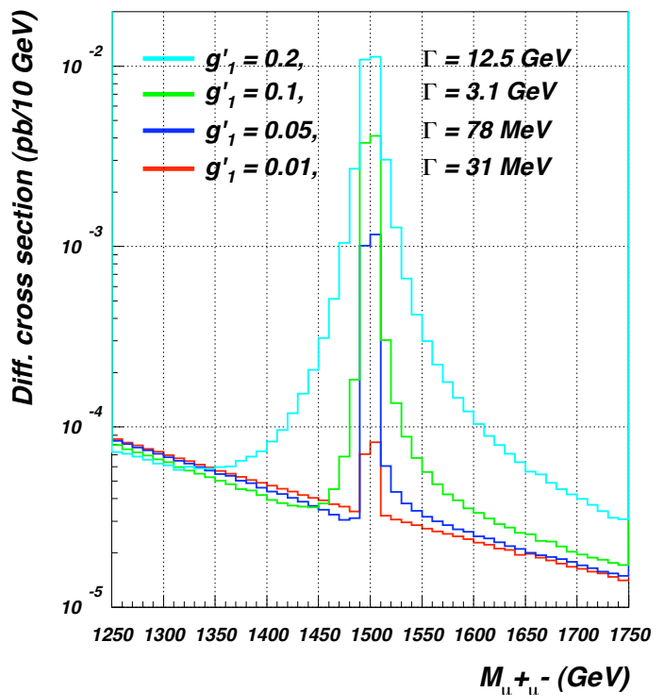


Hence minimum luminosity -> 0.5-5.0 x 10^{30} cm^{-2} sec^{-1}

for M(Z′) -> 1.5-5.0 TeV

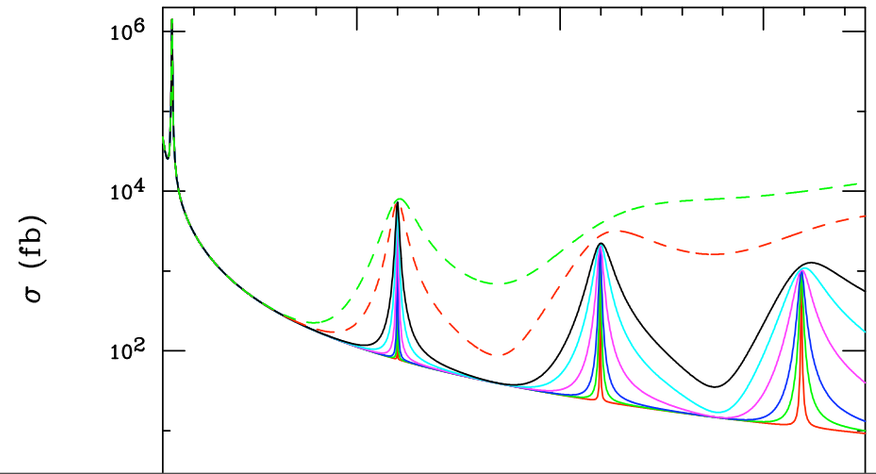
- Other new physics possibilities:
  - $Z_{(B-L)}$ L. Basso, A. Belyaev, S. Moretti and G. M. Pruna [arXiv:0903.4777v1]





$g'_1 = 0.20$: $\Gamma/M = 0.84$ %
$g'_1 = 0.10$: $\Gamma/M = 0.21$ %
$g'_1 = 0.05$: $\Gamma/M = 0.05$ %
$g'_1 = 0.02$: $\Gamma/M = 0.0084$ %

- scalars: h, H^0, A^0,... Large tan(β) -- resolving nearby states: H^0, A^0 in decoupling limit
- ED: KK modes -- Br($\mu^+\mu^-$) same as Z case
  - two parameters:
    - mass scale ∝ first KK mode;
    - width ∝ 5D curvature / effective 4D Planck scale.



- new strong dynamics: Bound state resonances
  resolve nearby states

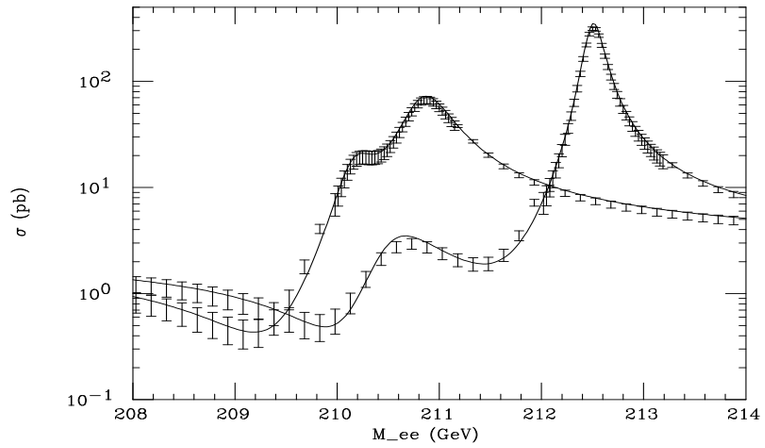


Figure 2: Cross sections for $\mu^+\mu^- \to \rho_T, \omega_T \to e^+e^-$ for $M_{\rho_T} = 210\,\text{GeV}$ and $M_{\omega_T} = 211\,\text{GeV}$ (higher-peaked curve) and 209 GeV. Statistical errors only are shown for resolutions and luminosities described in the text. The solid lines are the theoretical cross sections (perfect resolution).

- Fourth generation quarks - new QQ bound states.

  If $|V_{4i}| < 10^{-3}$:

$$B(\mu^+\mu^-) \approx 6\% \to R_{peak} \approx 10^4$$

Need fine resolution: $\Gamma/M \approx 5 \times 10^{-6}$

Abridged Parameter List

Machine	1.5-TeV $\mu^+\mu^-$	3.0-TeV $\mu^+\mu^-$	CLIC 3 TeV
$\mathcal{L}_{\text{peak}}$ [cm^{-2} s^{-1}]	7×10^{34}	8.2×10^{34}	8×10^{34} tot
$\mathcal{L}_{\text{avg}}$ [cm^{-2} s^{-1}]	3.0×10^{34}	3.5×10^{34}	3.1×10^{34} 99%
$\Delta p/p$ [%]	1	1	0.35
$\beta^\star$	0.5 cm	0.5 cm	35 μm
Turns / lifetime	2000	2400	
Rep. rate [Hz]	65	32	
Mean dipole field	10 T	10 T	
Circumference [m]	2272	3842	33.2 km site
Bunch spacing	0.75 μs	1.28 μs	0.67 ns

- What is the optimum search strategy for low luminosity?

- Feasibility of 10 TeV at low luminosity?