Towards End-to-end 6D cooling Simulation

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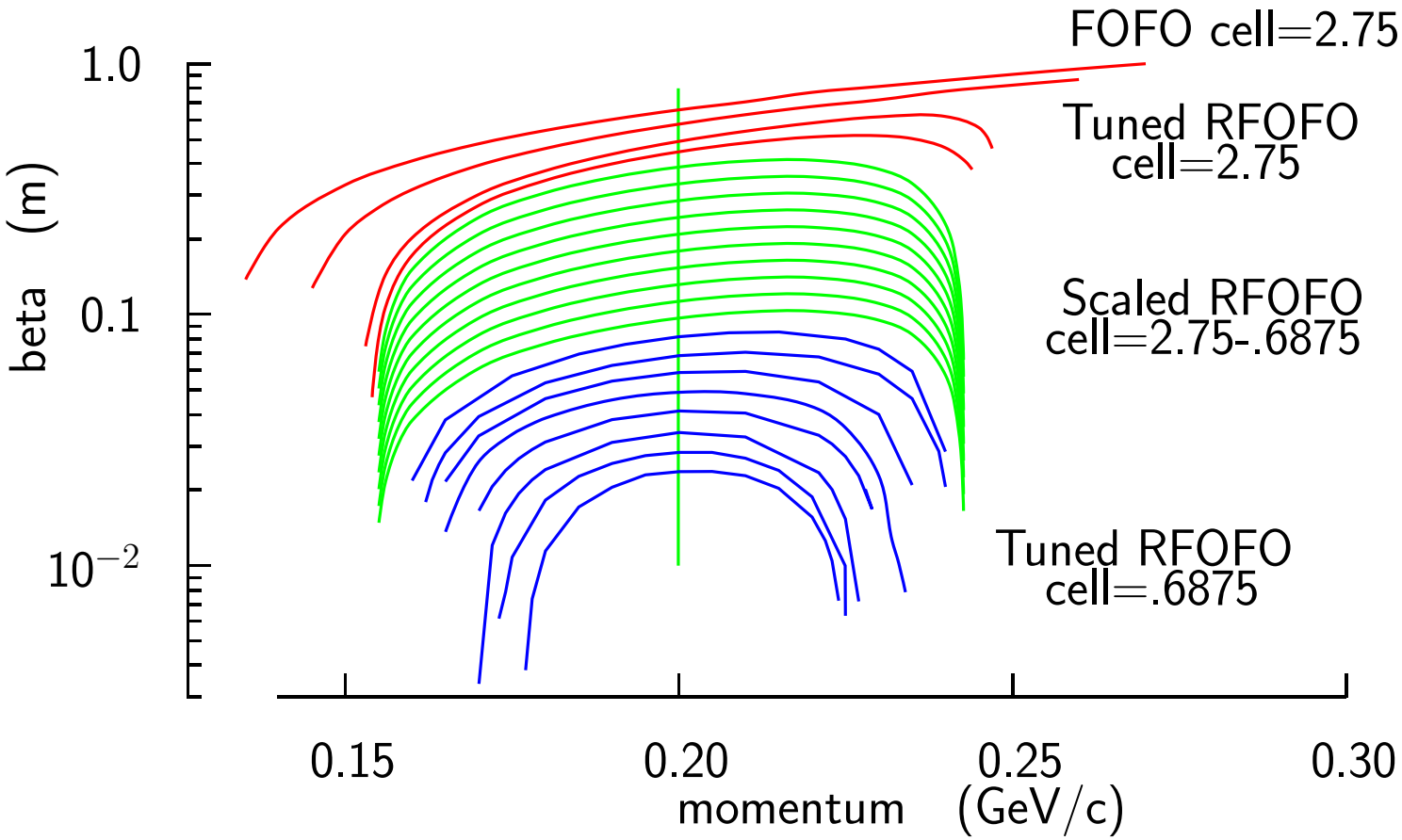
Thursday

4/14/10

- Tapered lattices
- Application to Neutrino Factories
- Application to Muon Colliders
  - Before Merge
  - New 6D Merge
  - After Merge
- Transmission
- Conclusion
- Appendix on parameters

Tapered lattices

Beta can also be reduced by lowering all dimensions and raising the axial fields
We choose to reduce cells from 275 cm to 68.75 cm,
With axial fields rising from $\approx$ 3T to $\approx$ 12 T



- Range of betas from 68 cm to 2.37 cm (29:1) as required
- 22 stages with betas decreasing by 15%/stage

Simulation using matrix emittance exchange

- Full simulation of tapered Guggenheim requires
  - coil tilting for dipole fields
  - wedge absorbers
  - and is time consuming
- But required dipole field $<<$ solenoid fields (eg: .125 vs 3 T)
  - focusing betas are almost identical to those in a linear channel
  - resulting emittance exchange is close to ideal
- It is easier and faster to simulate a linear channel
- Adding exchange by a matrix acting on (x, x', y, y', σ_z, σ_p/p)

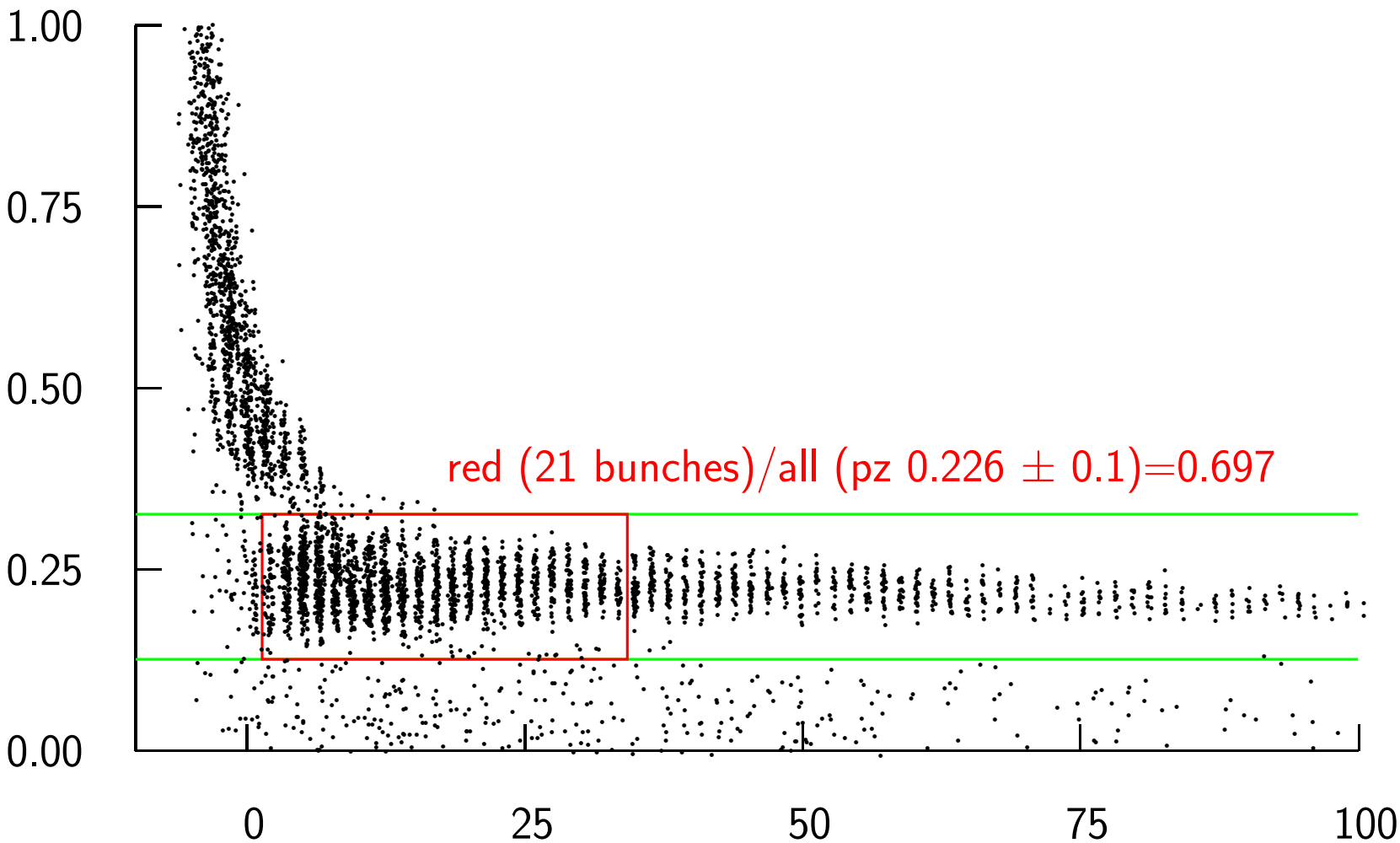
$$\begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1+\delta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\delta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 - 2\delta \end{matrix}$$

- Comparisons with full simulation of an un-tapered lattice are close

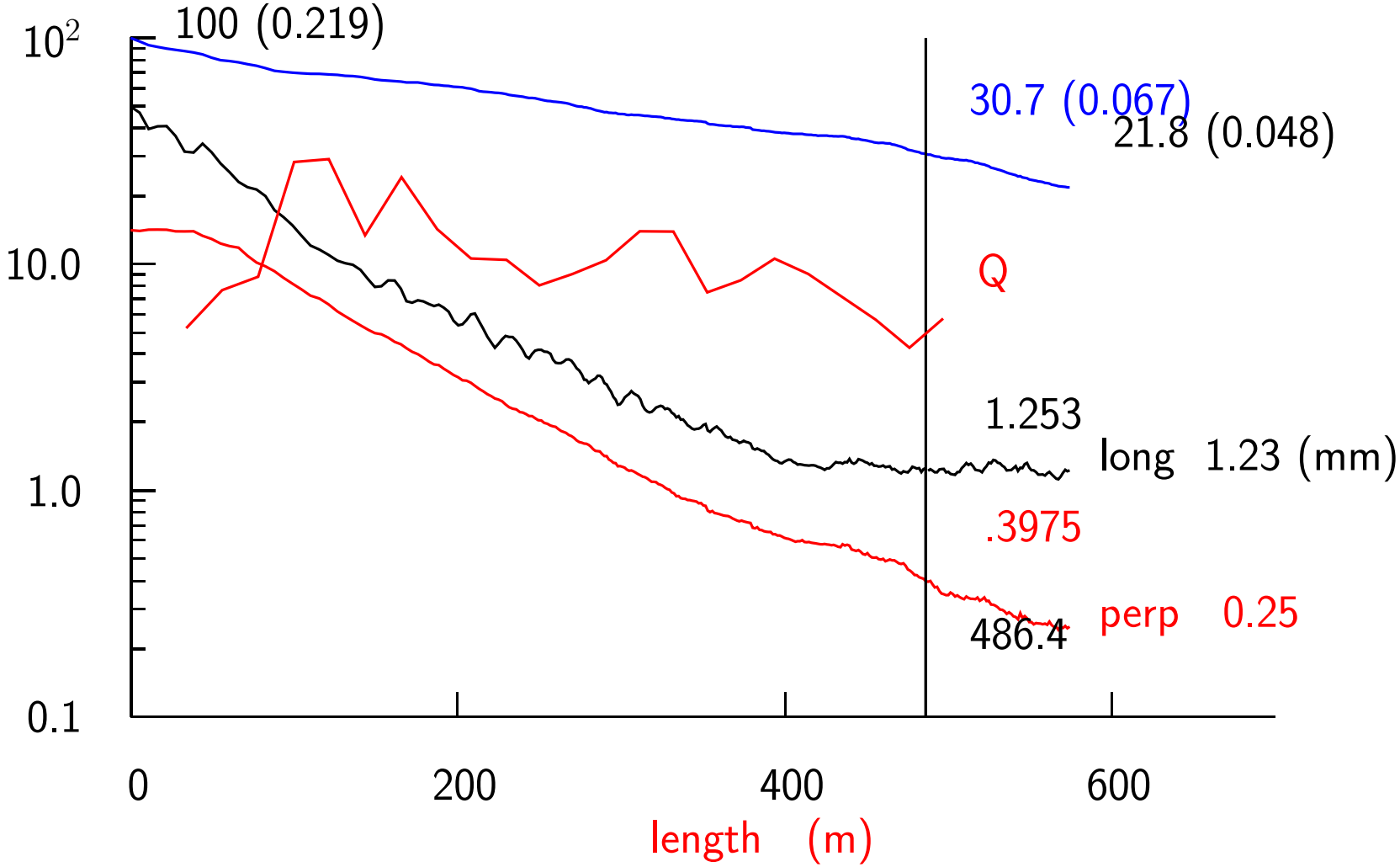
Input for Simulation

ICOOL Simulation of phase rotation from MARS 15 and 8 GeV protons

- MARS 15 400,00 $\times \frac{50}{93.1}$ =214,800 p giving 71831 muons $\rightarrow \mu/\mathrm{p} = 0.334$
- Selecting momenta= 226 $\pm$ 100 MeV/c gives 46,963 muons $\rightarrow \mu/\mathrm{p} = 0.219$

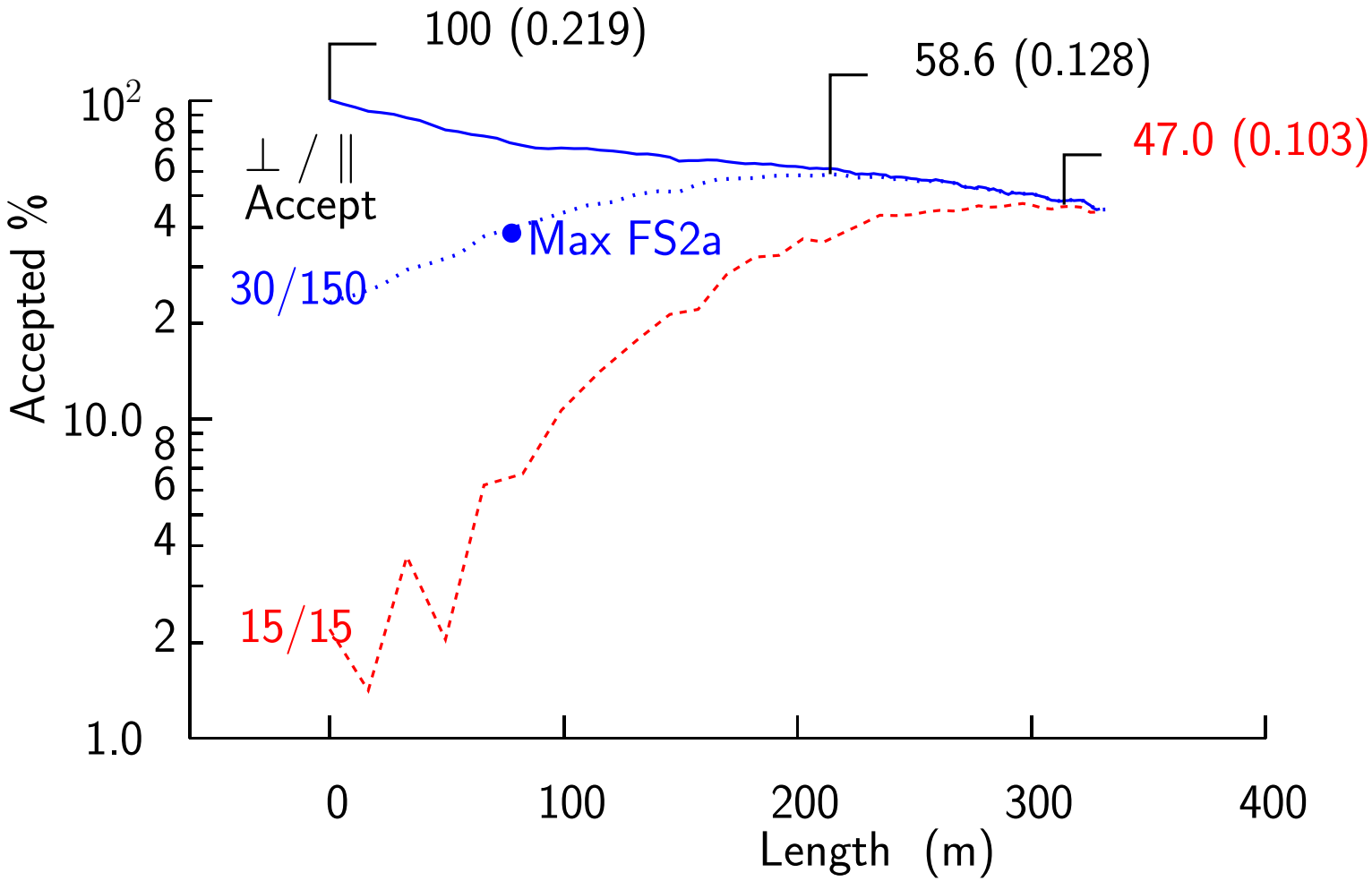


ICOOL simulated emittances vs length without merge



- No transverse emittance growth at matches is observed
- Initial Q is better than in un-tapered lattice (23 vs. 15)
- Final Q is better than in un-tapered lattice (12 vs. 8)

Application to Neutrino Factory



- Max into 30 mm trans and 150 mm long = 0.128 μ/p (1.6 × ISS 0.08)
  But cooling longer (80→160 m)
- Max into 15 mm trans and 15 mm long = 0.103 μ/p (1.28 × ISS 0.08)
  Acceleration and storage rings now less expensive
  but Cooling (→ 240 m)

Application to Muon Collider

1) Cooling before Merge

- Initial cooling in a linear lattice is less efficient than 6D cooling
- But initial Guggenheim needs charge separation straight after phase rotation
- This is very hard because of the large emittances

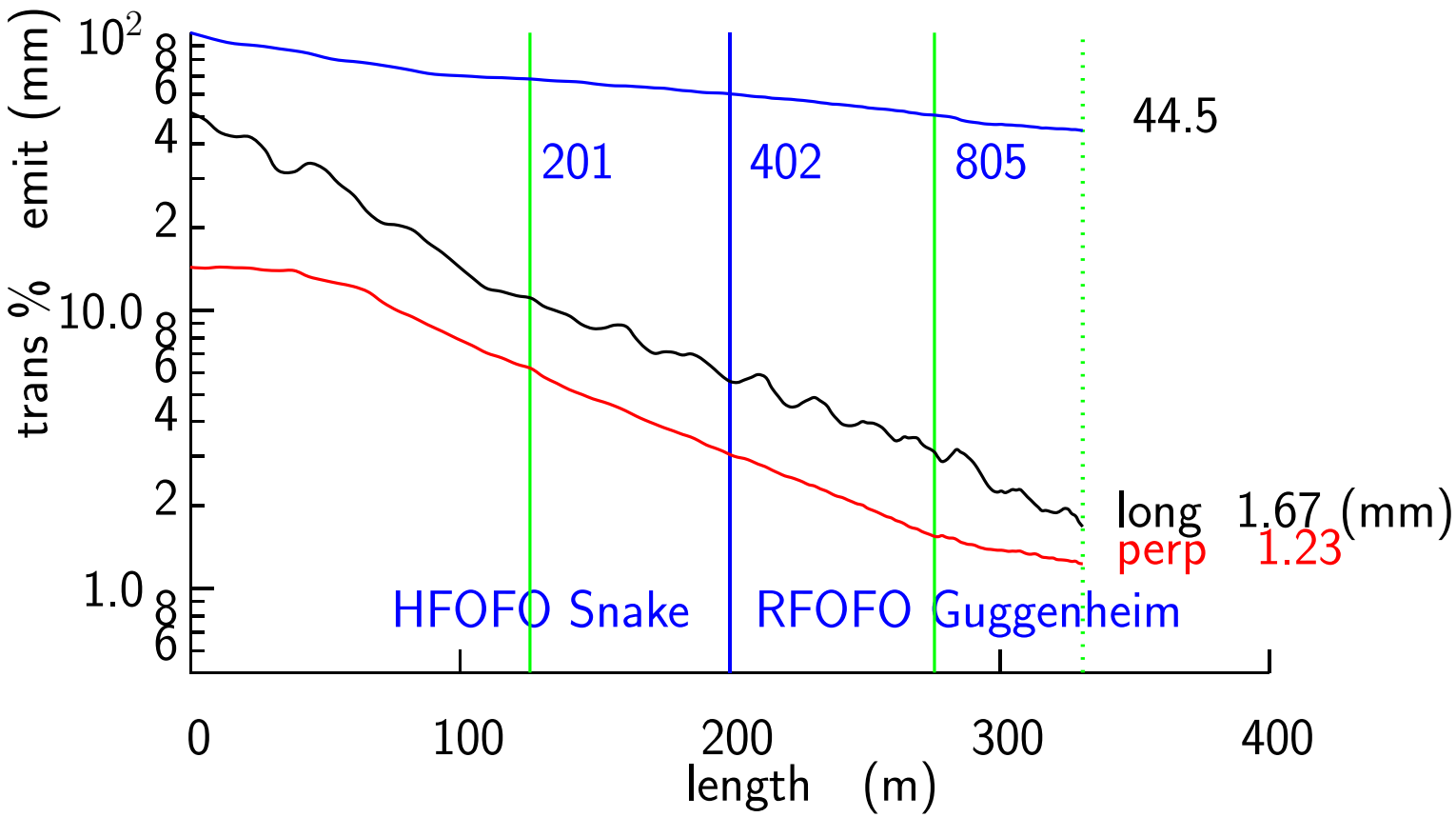
- So Alexahin's HFOFO snake is assumed for the early part:
- We have modified coil dimensions in a 1/2 scale snake

	r1	r2	$\beta_\perp$	Bmax	j	T	$\epsilon_\perp$
	cm	cm	cm	T	A/mm^2	MPa	mm
Yuri's	60	92	96	6.4	53	203	11
1/2 scale	20	56	48	7.8	88	132	5.5

- We are assuming that the snake, when tapered, will perform as well as these Guggenheims
- Charge separation would then be after the FOFO snake
- I will assume a separation transmission of 85 %

ICOOL Simulation of Guggenheim

- Increase length of stage 14 to 80 cells
- Increase δ in this stage from 0.076 to 0.095
- Skip all later stages



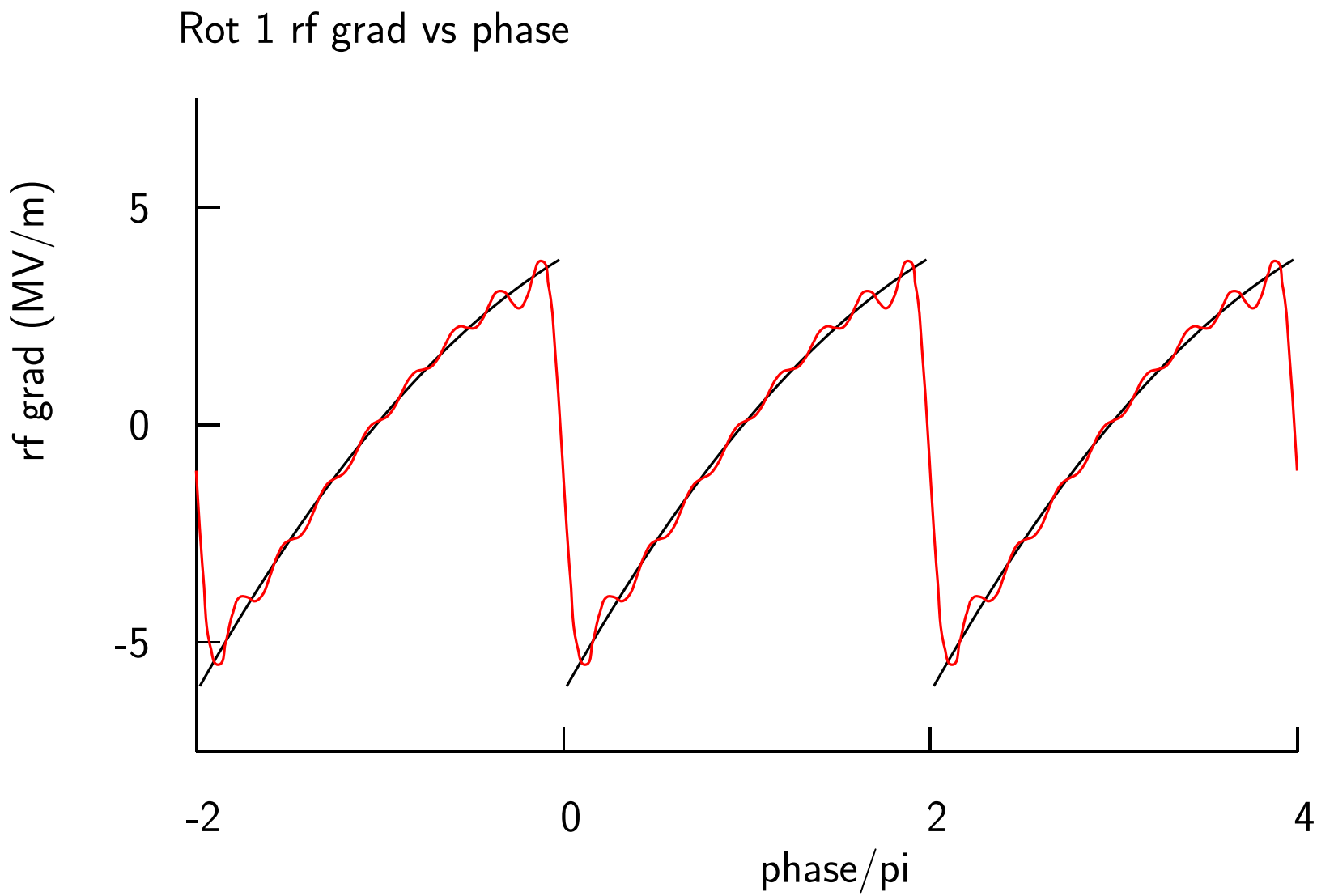
482.3

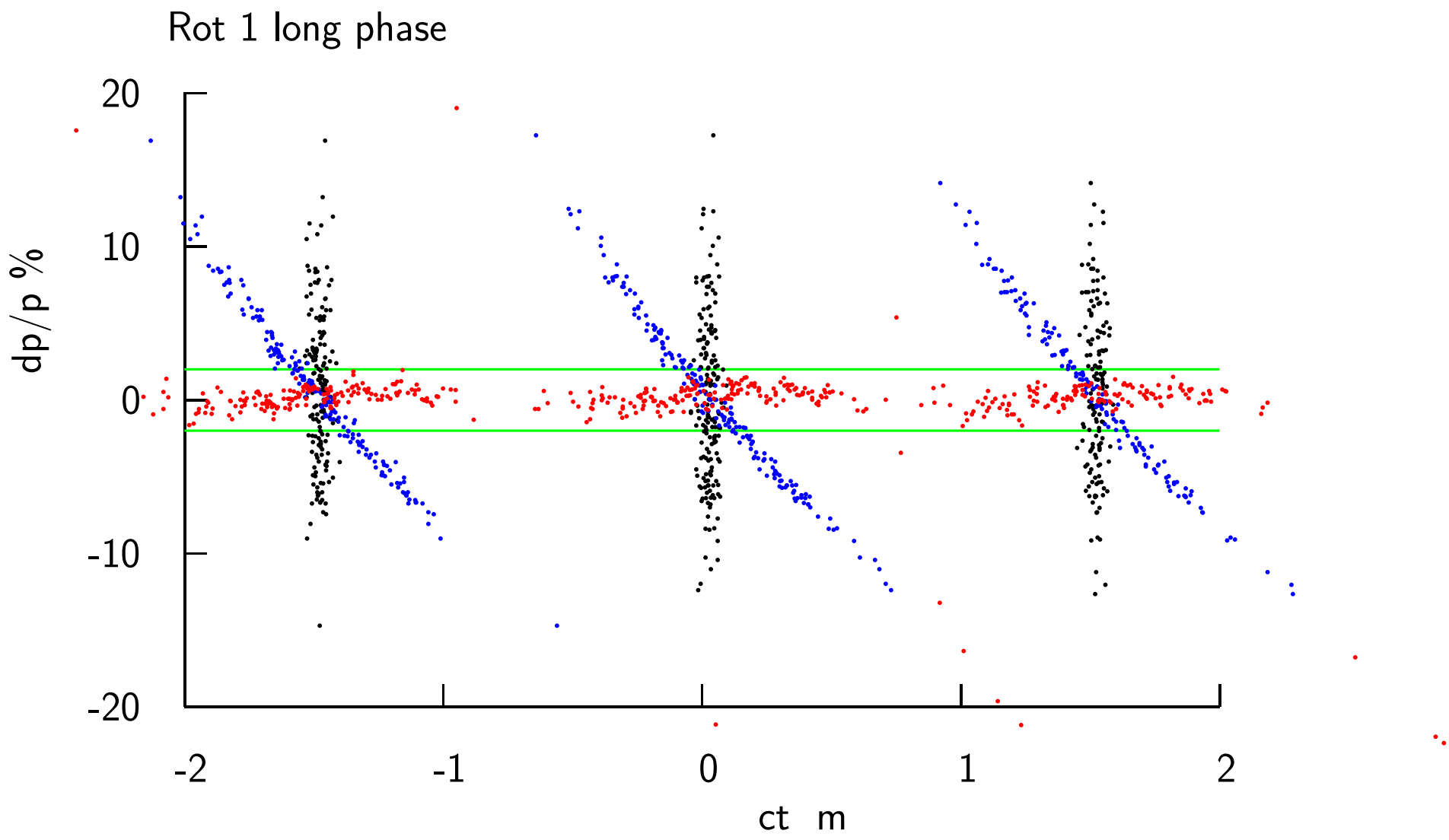
New 6D Merge

- My previous merge was in a single dimension.
- It stacked 21 bunches in momentum space, assuming a negative k FFAG and an arbitrarily good induction linac pulse shape
- It had transmission only of order 70%, and was of questionable practicality

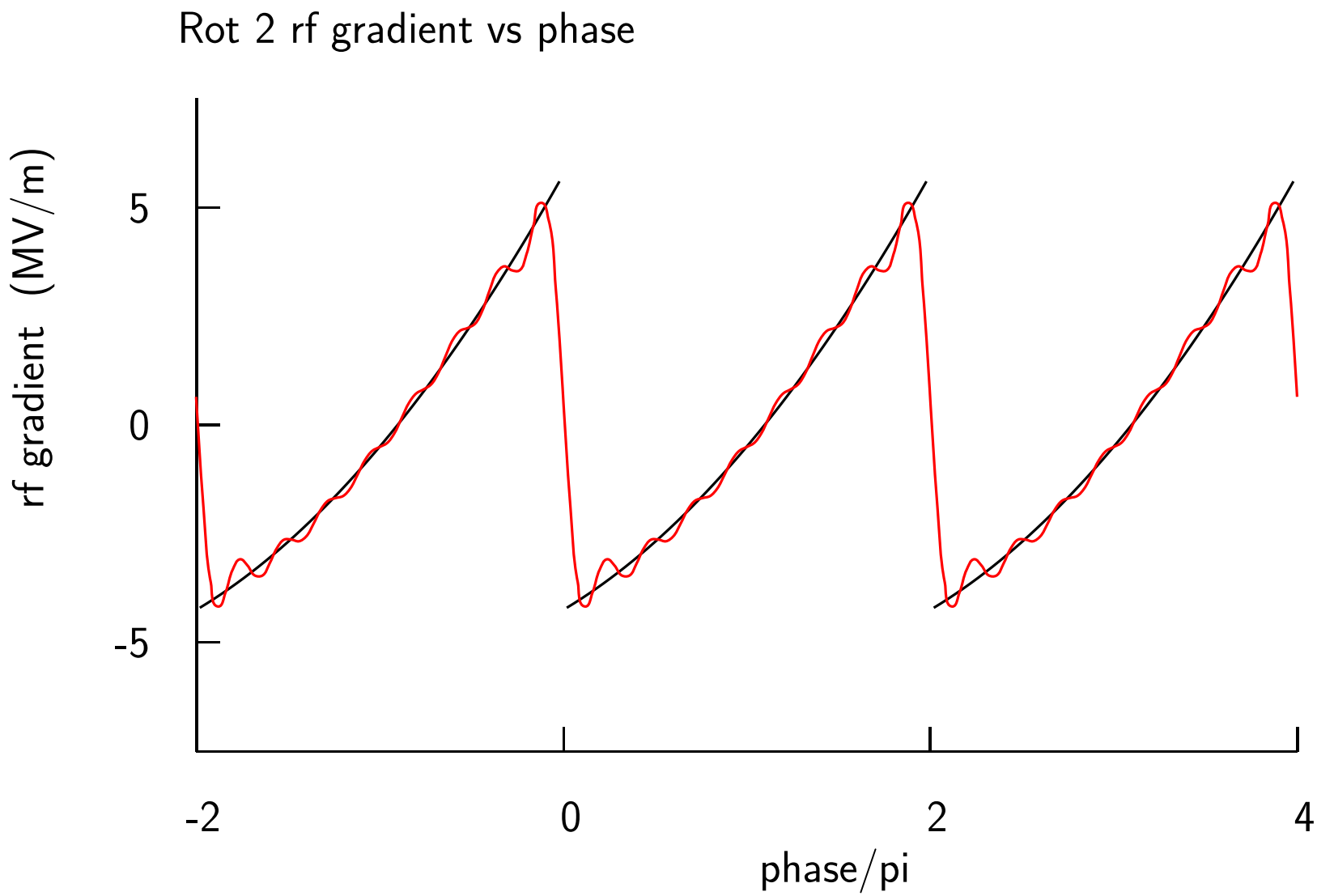
- The new merge uses all 6 dimensions: longitudinally stacking each group of 3 bunches into one, and then combining 7 of these stacks in the xy plane
- Merging in the longitudinal plane is done, as before, by a double phase rotation:
  - The first rotation lowers the momentum spreads, makes the bunches longer, and thus closer together
    it uses 201 MHz with harmonics and simple drifts
  - The second rotation brings each group of three to be above one another in momentum
    it uses 67 MHz rf with harmonics and a wiggler drift
- Merging in the transverse plain is performed using an Ankenbrandt trombone and a funnel into a hexagon pattern in xy

Long Rotation # 1 using 201 MHz rf plus harmonics

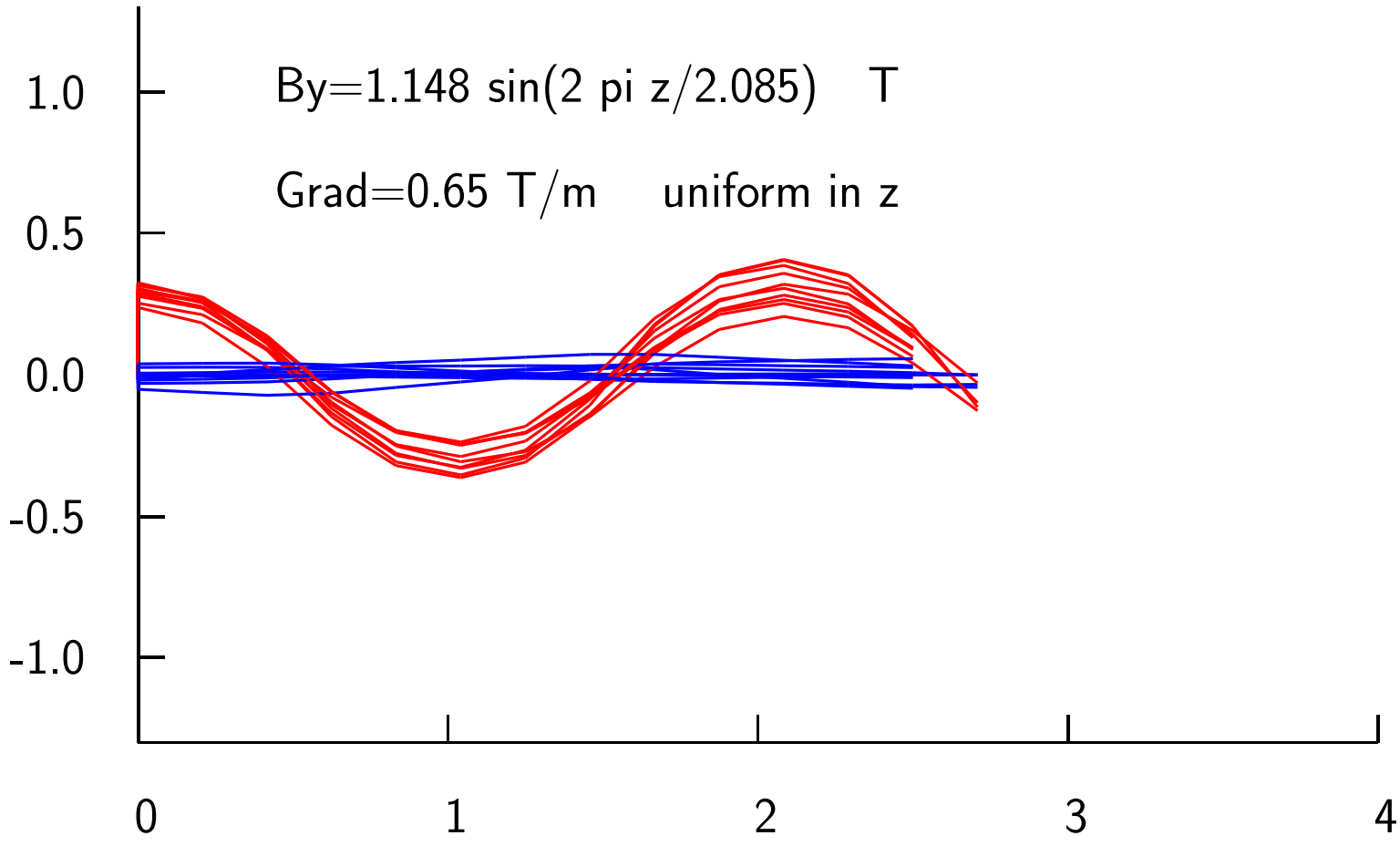


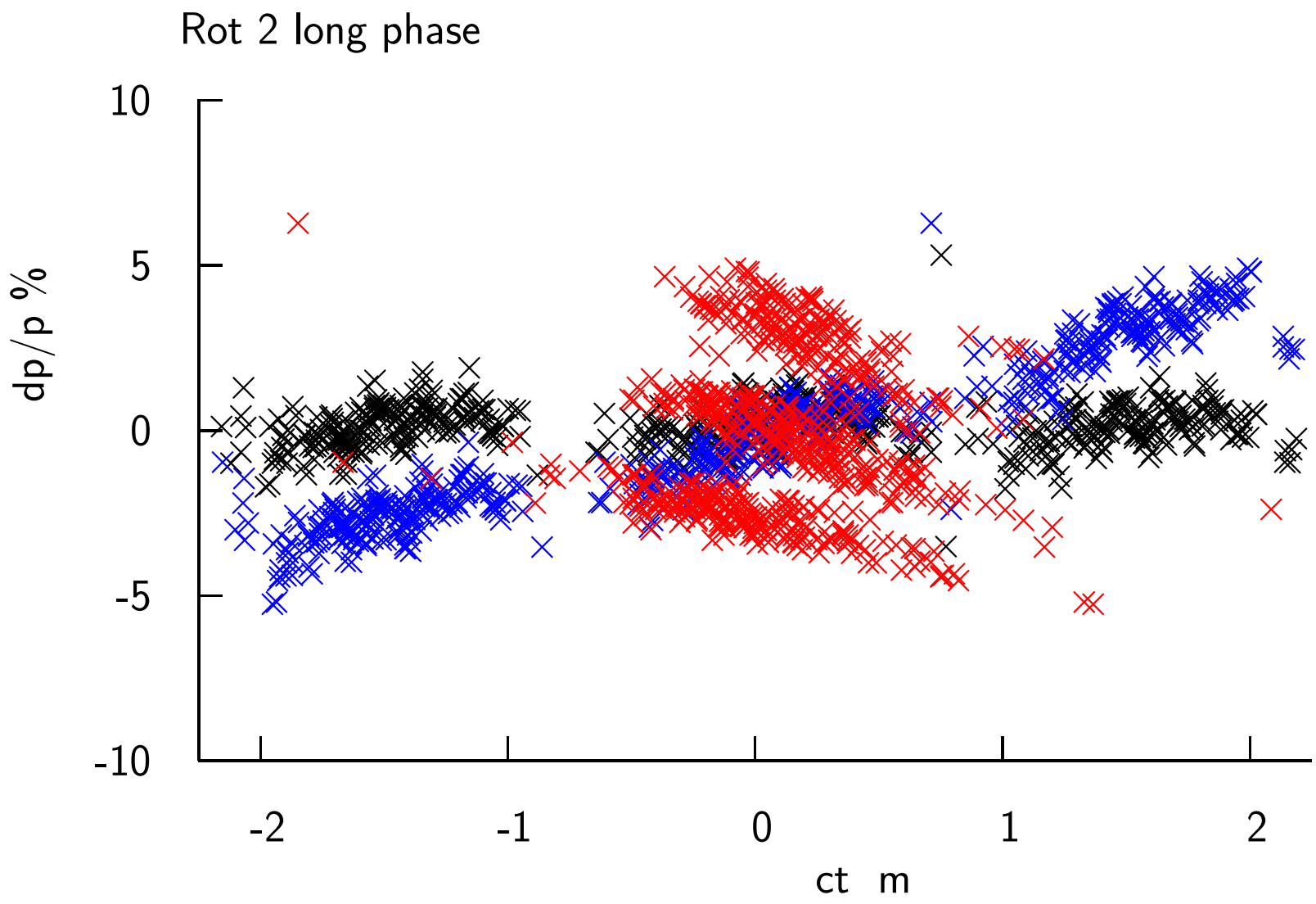

Rot 1 long phase
20
10
0
-10
-20
dp/p %
-2
-1
0
1
2
ct m

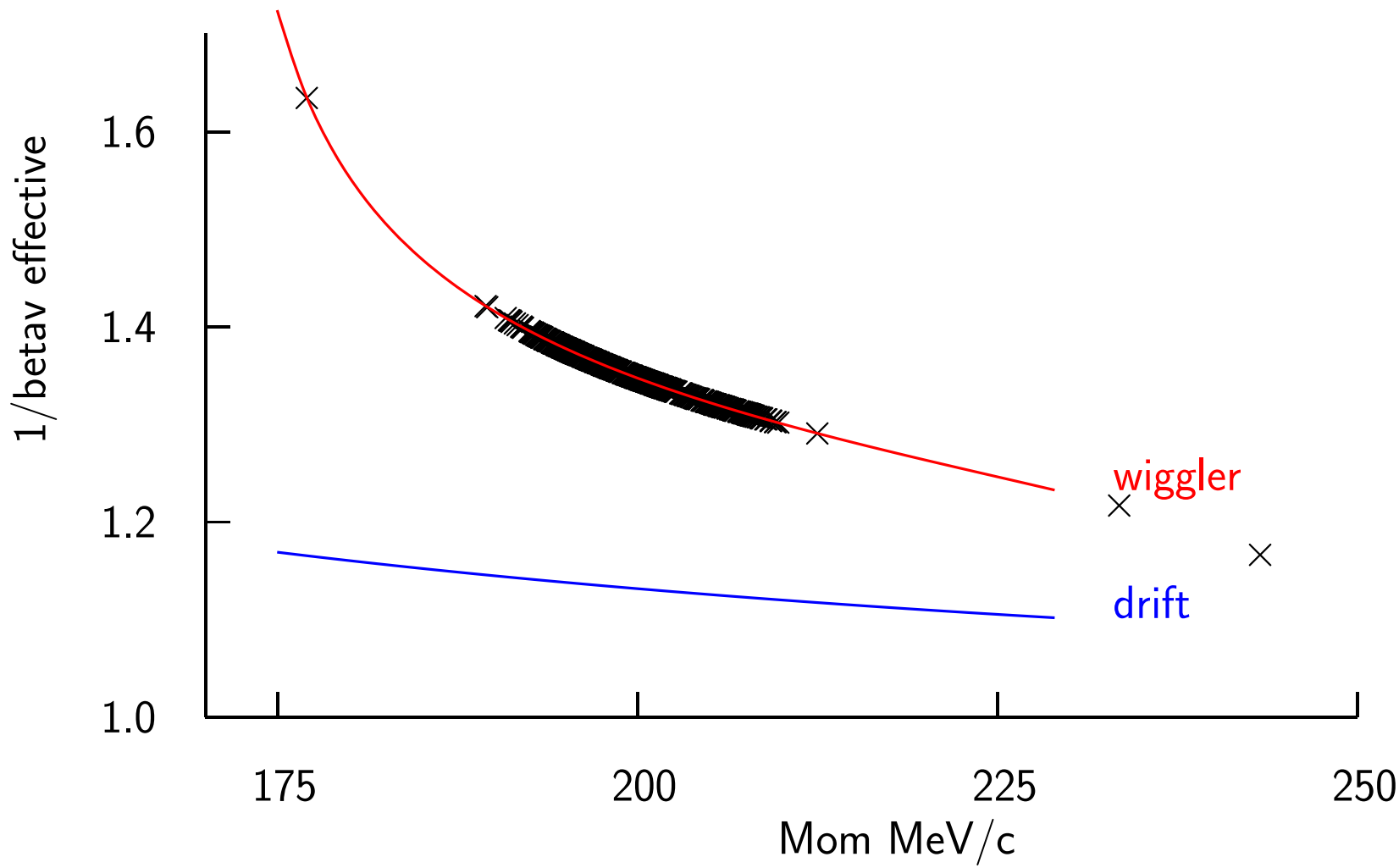
Long Rotation # 2 using 67 MHz plus harmonics



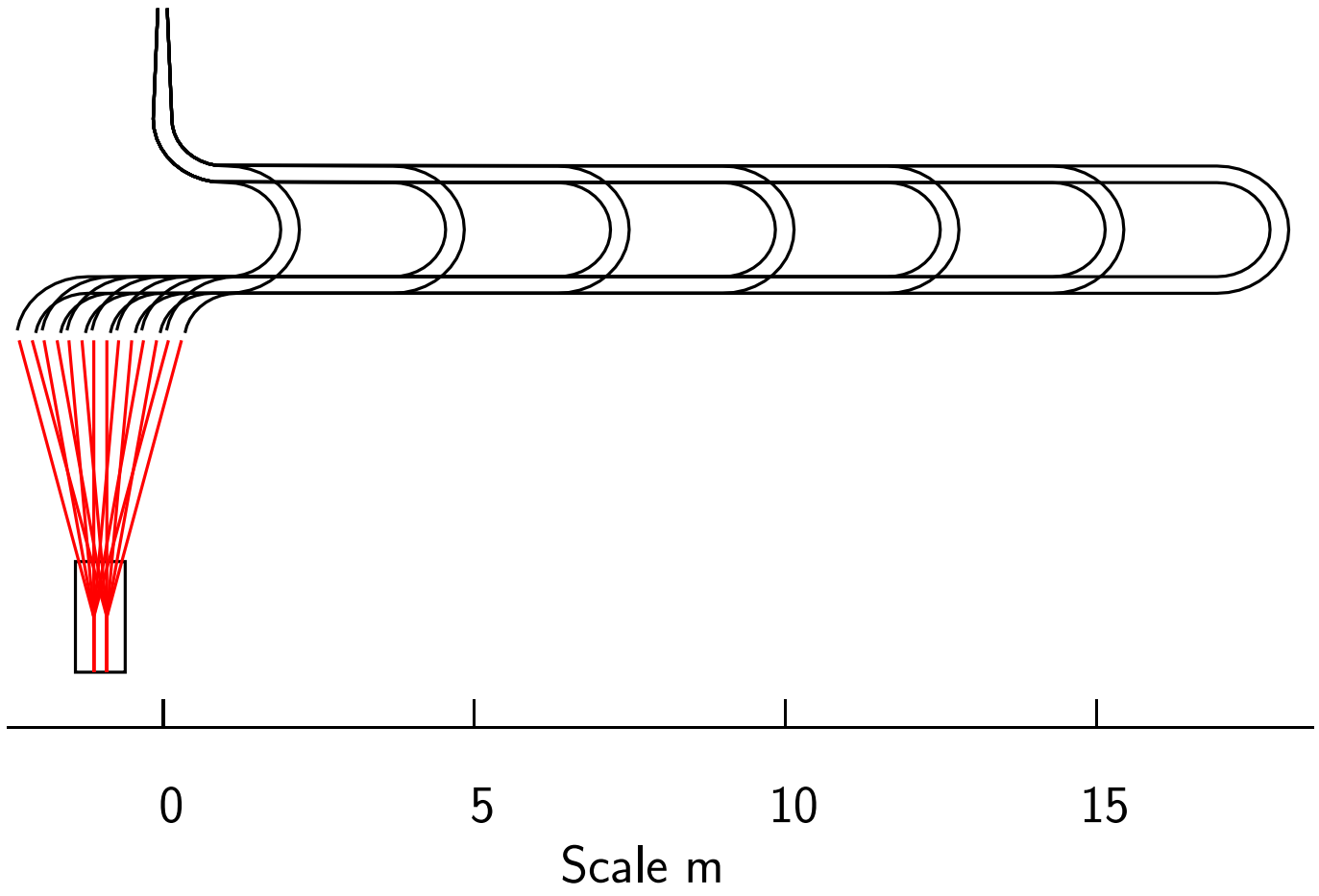
Planar wiggler

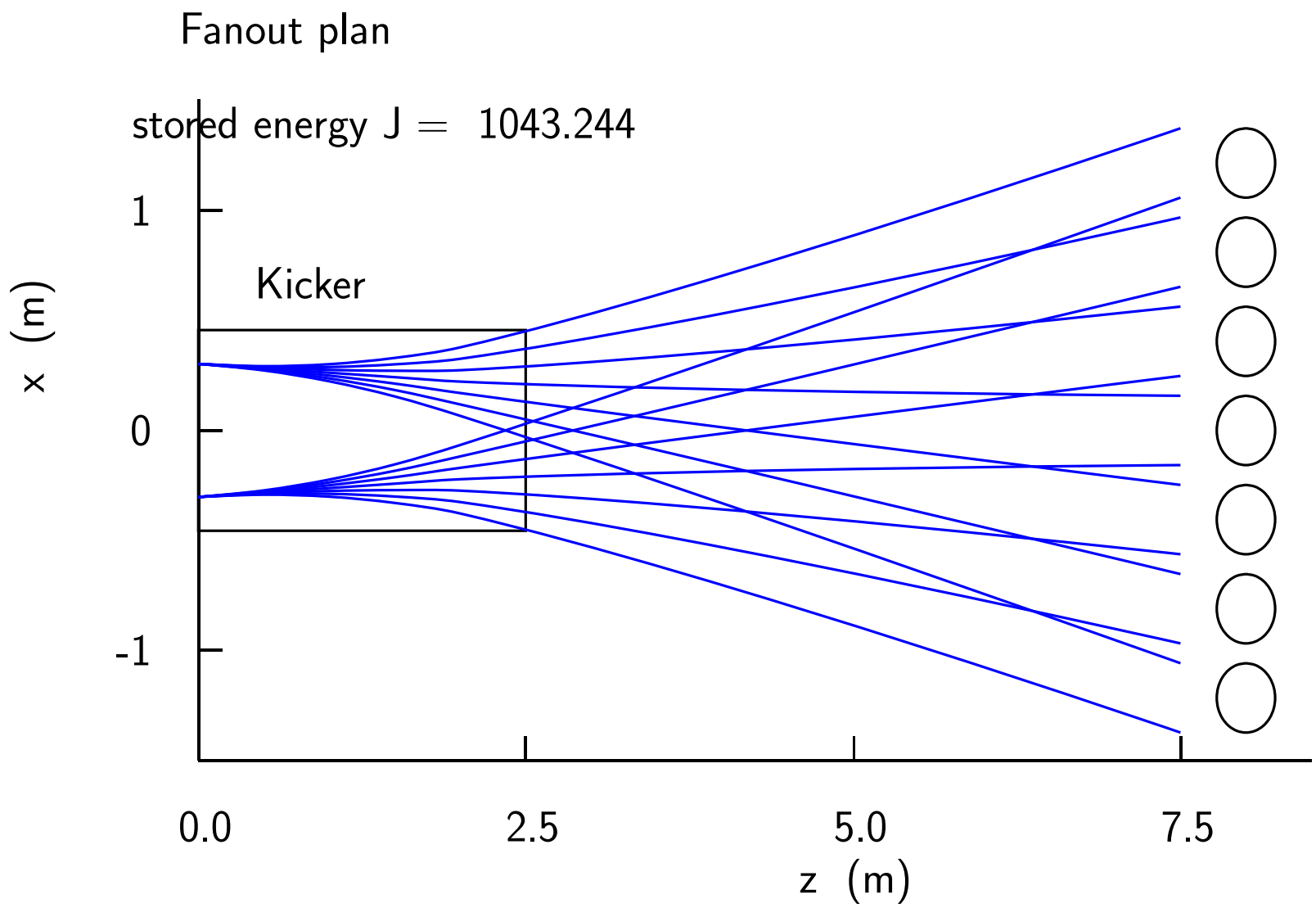


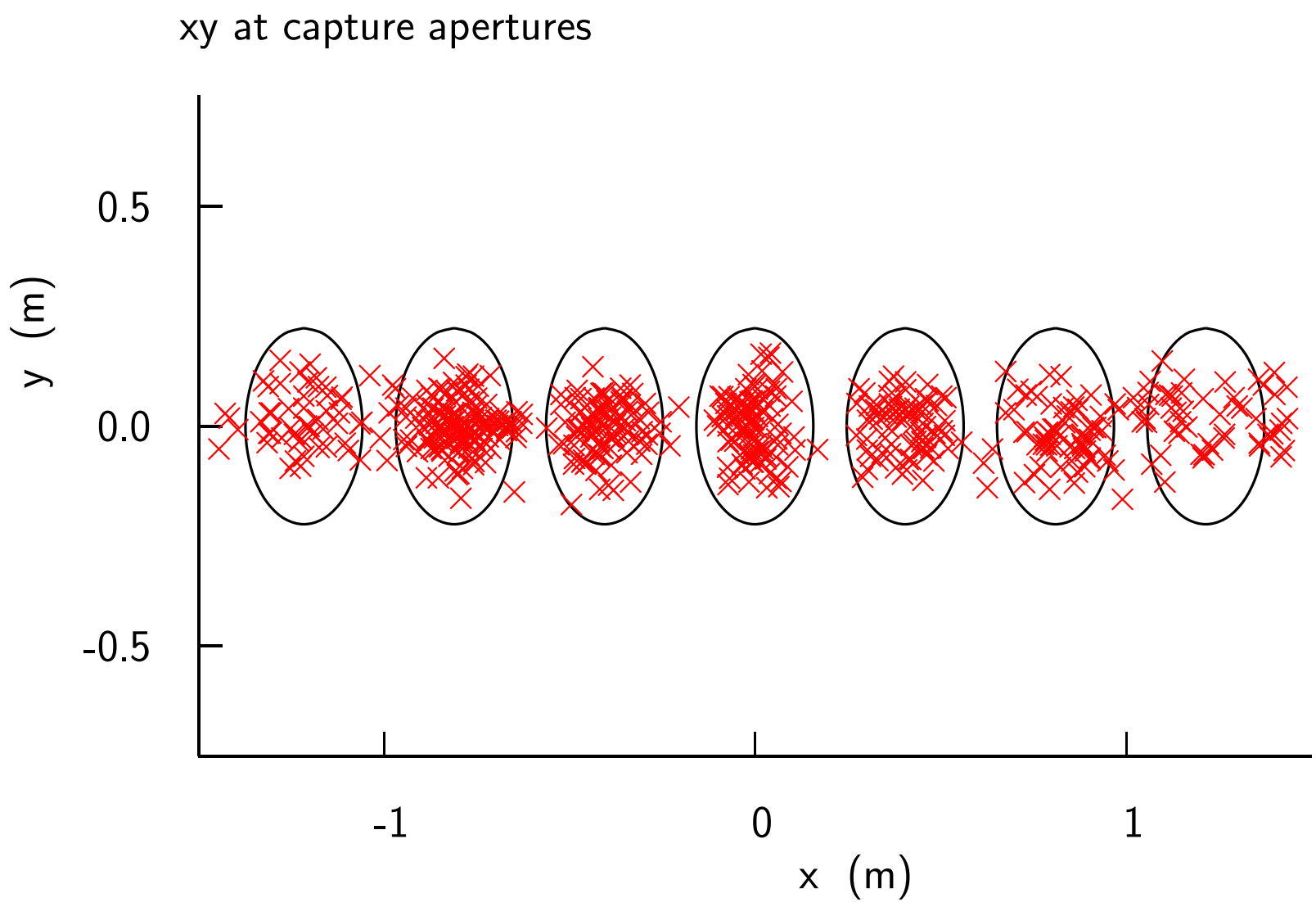

Rot 2 long phase
10
5
0
-5
-10
dp/p %
-2
-1
0
1
2
ct m

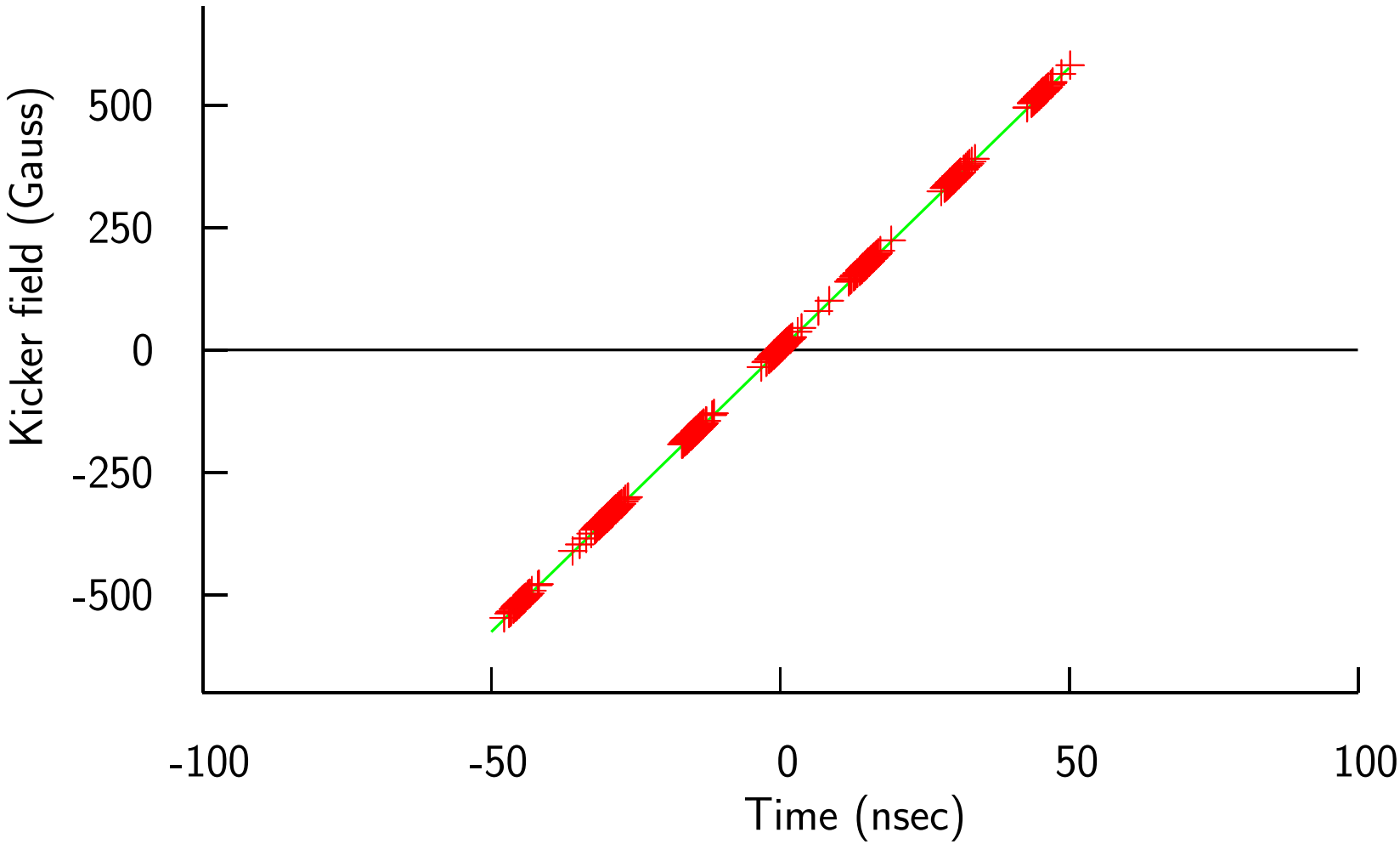

1/betav effective
1.6
1.4
1.2
1.0
175
200
225
250
Mom MeV/c
wiggler
drift

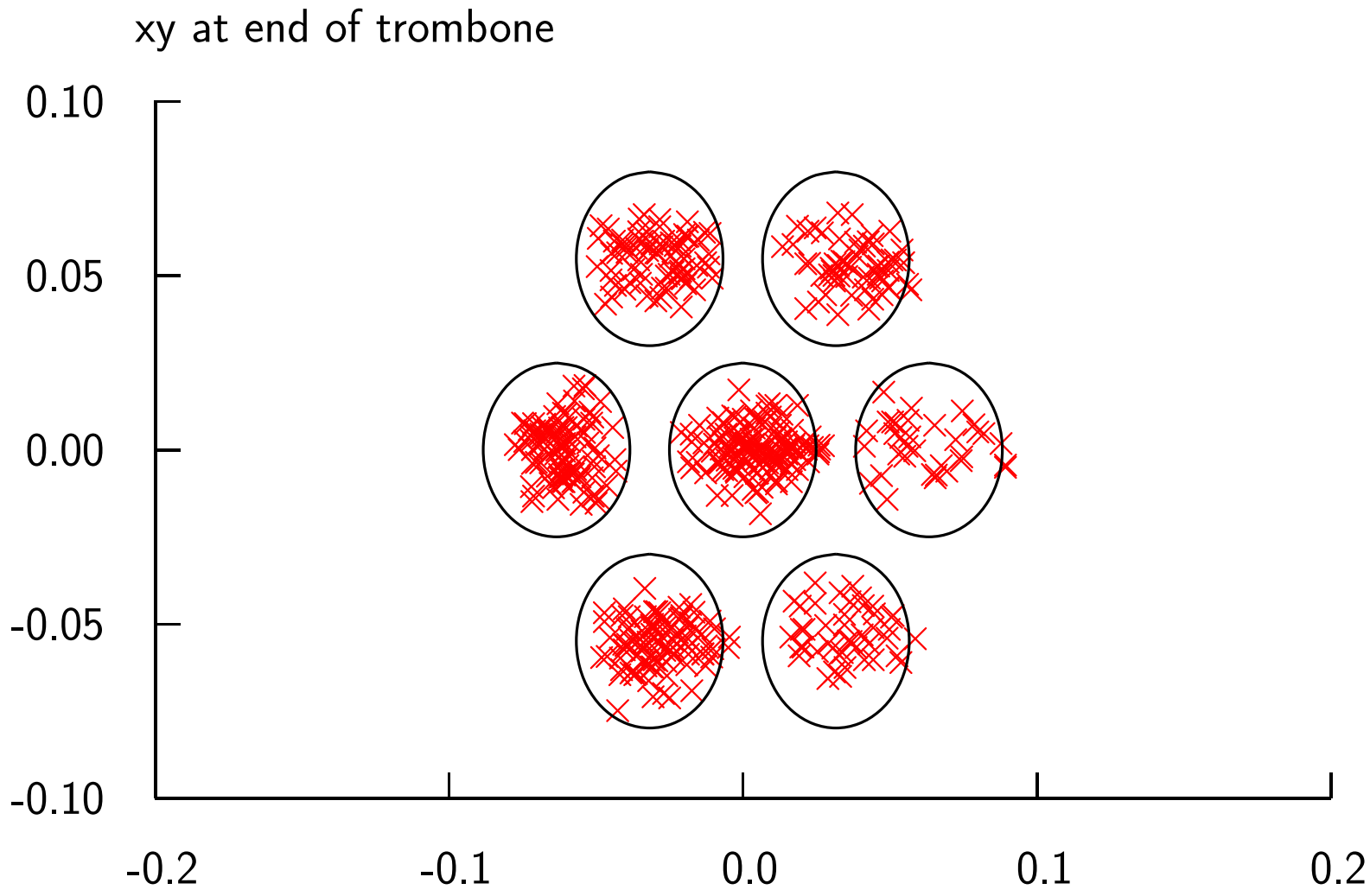
Transverse stack

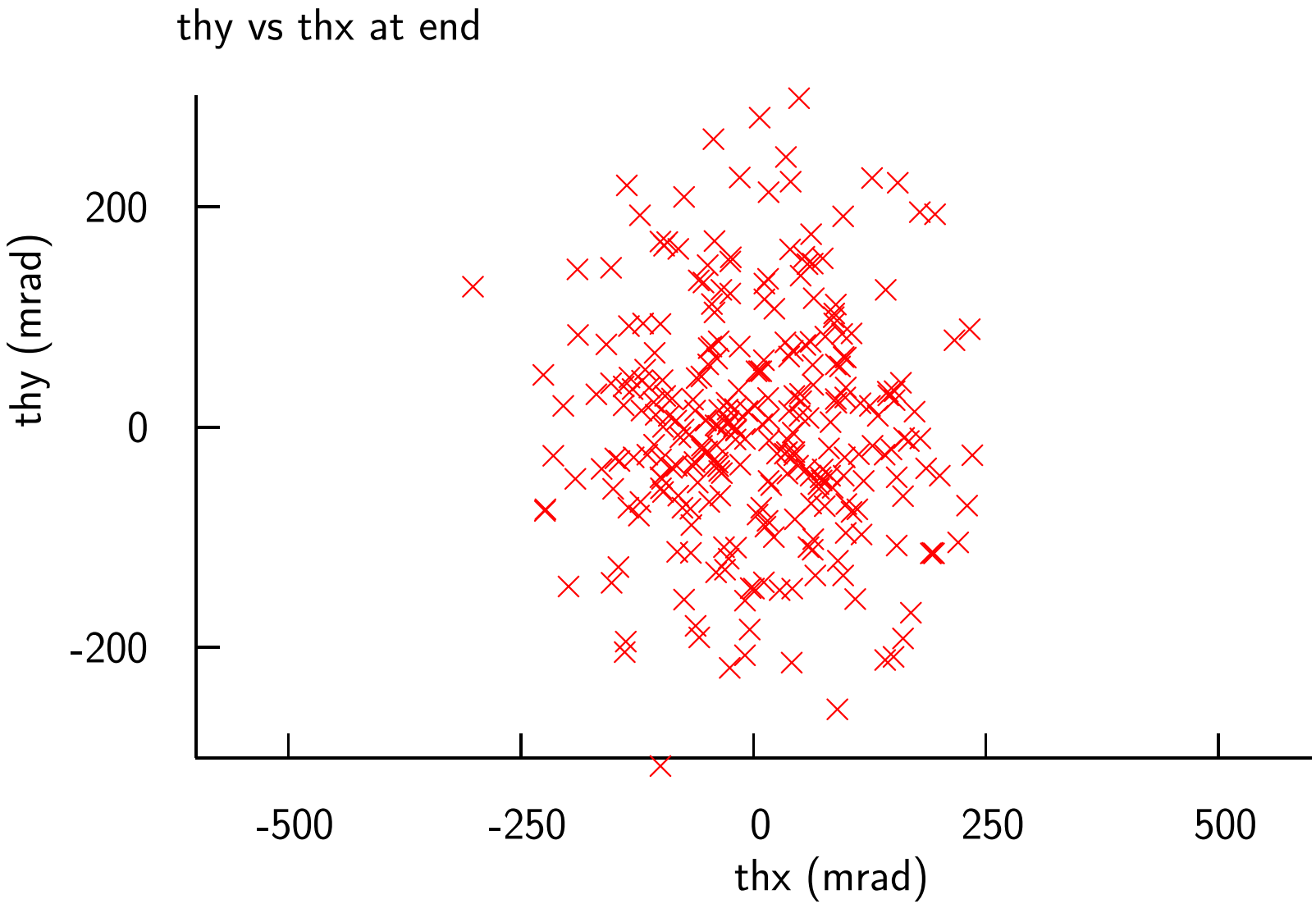



Fanout plan
stored energy J = 1043.244
Kicker
1
0
-1
x (m)
0.0
2.5
5.0
7.5
z (m)

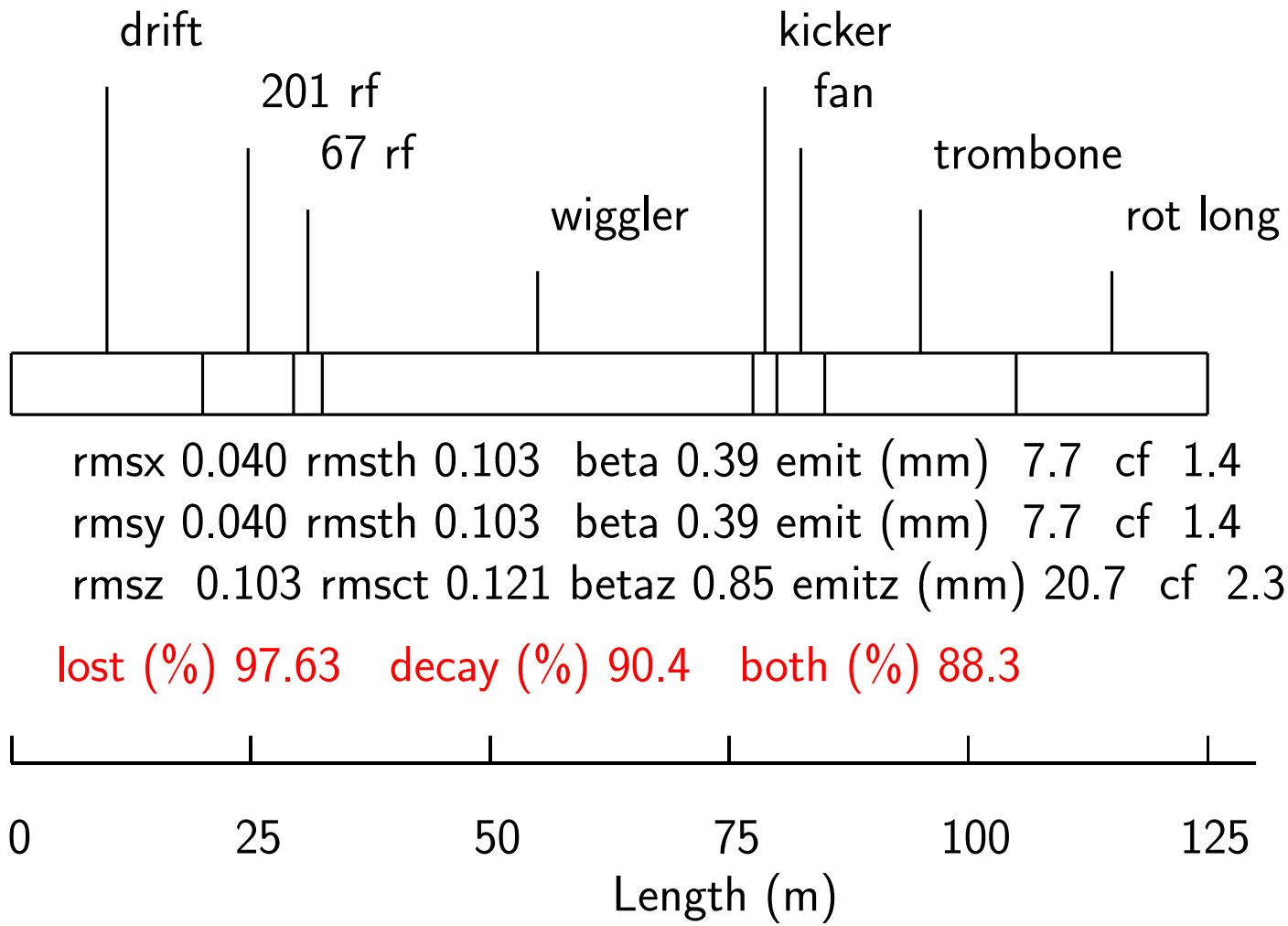

xy at capture apertures
0.5
0.0
-0.5
y (m)
-1
0
1
x (m)


Kicker field (Gauss)
500
250
0
-250
-500
-100
-50
0
50
100
Time (nsec)


xy at end of trombone
0.10
0.05
0.00
-0.05
-0.10
-0.2
-0.1
0.0
0.1
0.2


thy vs thx at end
thy (mrad)
200
0
-200
-500
-250
0
250
500
thx (mrad)

Section lengths

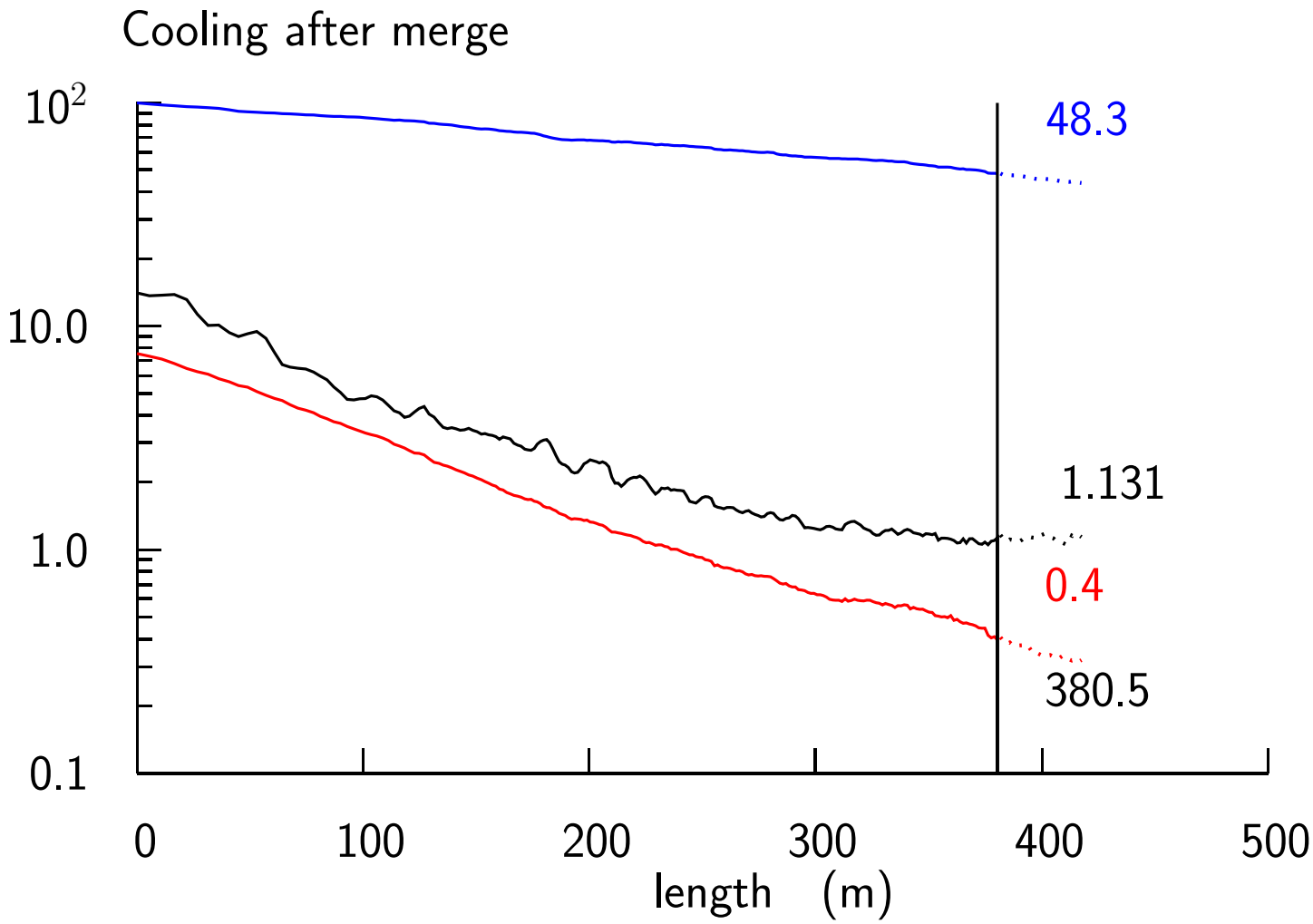


rmsx 0.040 rmsth 0.103 beta 0.39 emit (mm) 7.7 cf 1.4
rmsy 0.040 rmsth 0.103 beta 0.39 emit (mm) 7.7 cf 1.4
rmsz 0.103 rmsct 0.121 betaz 0.85 emitz (mm) 20.7 cf 2.3

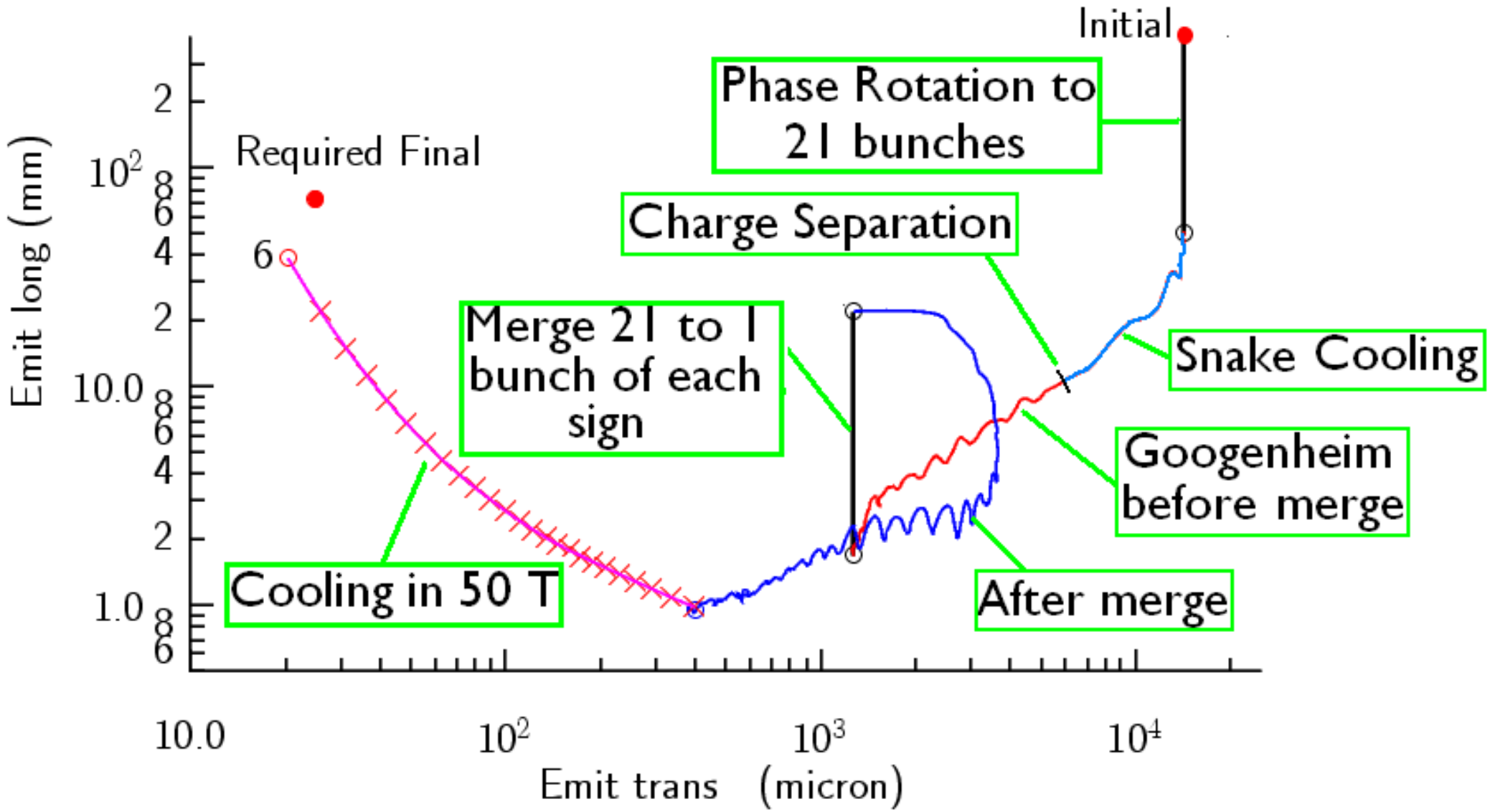
lost (%) 97.63 decay (%) 90.4 both (%) 88.3

3) Cooling after Merge

- less initial long emittance after merging
- Skip first 4 stages

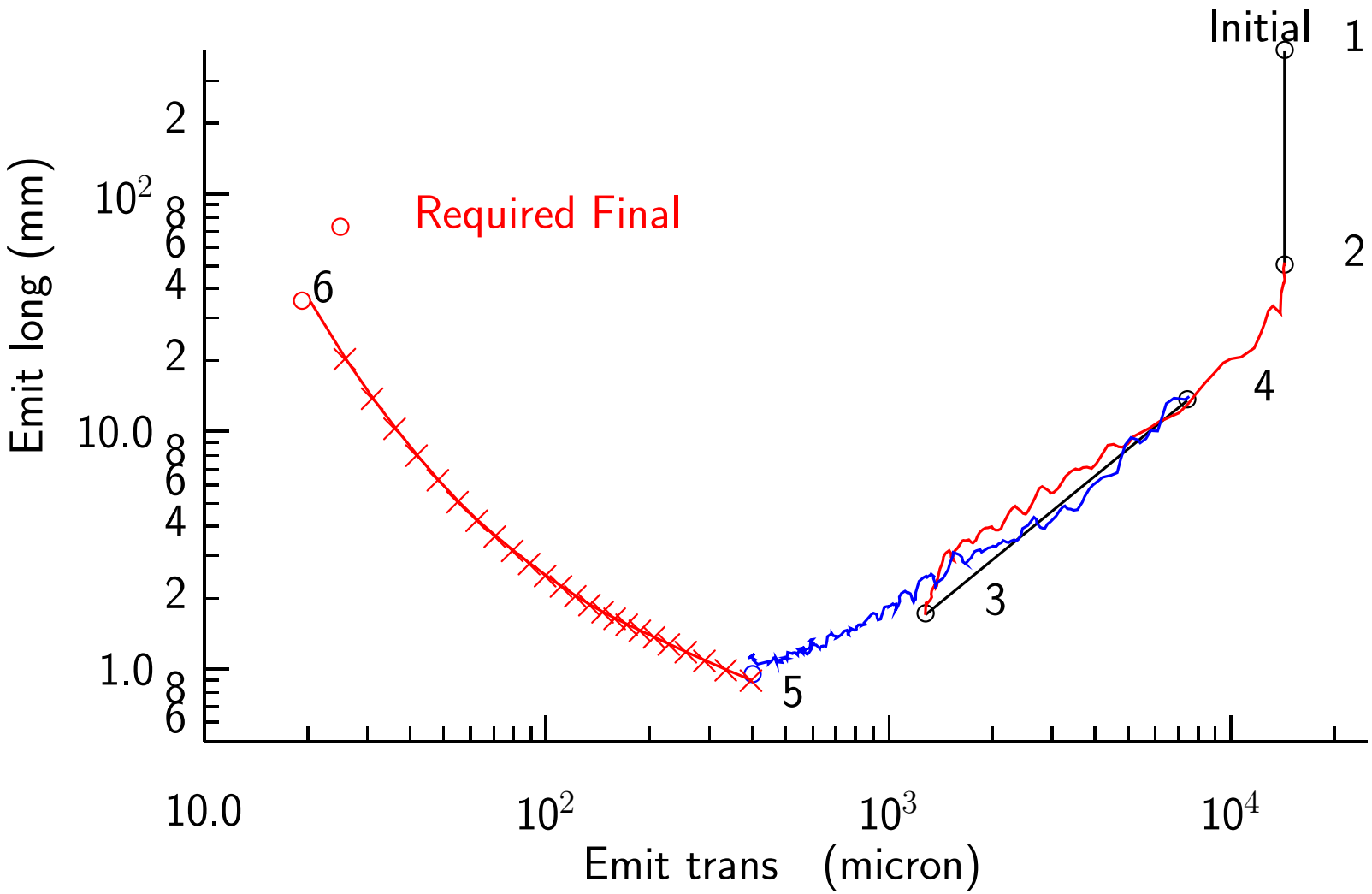


Old Long vs. Transverse emittances



- Note transverse heating after bunch merge

New Long vs. Transverse emittances



Transmission

	old	transmission	cumulative	mu/p
After rotation				0.334
Momenta = 226 ± 100 MeV/c		0.654	1.0	0.219
Best 21 bunches	(0.7)	0.7	0.7	0.153
Charge separation	(1.0)	0.85	0.59	0.129
6D Cooling before merge	(0.5)	0.468	0.28	0.061
Merge	(0.7)	0.88	0.25	0.055
6D Cooling after merge	(0.5)	0.48	0.12	0.026
50 T Cooling	(0.7)	0.7	0.08	0.018
Acceleration	(0.7)	0.7	0.06	0.013

- Transmission less than previous (7→6.0 %) from charge separation
- But initial production is better, from 8 GeV and MARS 15

For 2 10^{12} muons 1.54 10^{14} protons/bunch
Power at 12 Hz:

$$12 \times 1.5410^{14} \times 810^{9} \times 1.610^{-19} \;=\; 2.36 \text{ MW}$$

- Power is less than previous (4→2.36 MW)

Conclusion

- FOFO and RFOFO lattices designed for betas from 66 to 2.4 cm
- Simulation for Neutrino Factory using matrix emittance exchange
  - 1.6 $\times$ flux with same acceptances, but longer cooling (80$\rightarrow$160 m) GOOD
  - 1.28 times flux with only 15 mm (vs.30) transverse and 15 mm (vs. 150) longitudinal acceptances, but even longer cooling ($\rightarrow$ 240 m)
- Simulation for Muon Collider using matrix emittance exchange
  - Final transverse emittance less than specified for 50 T start GOOD
  - Final long emittance as specified for 50 T start GOOD
  - Transmission somewhat less than previous estimates
  - But production at 8 GeV and MARS 15 better than previous estimates
  - Required 8 GeV power is down (4$\rightarrow$2.8) GOOD
- 6D Merge more realistic and better than 1D longitudinal merge GOOD and removes wasteful transverse heating in following 6D cooling
- This is the first unoptimized 6D merge I can surely improve it
- This matrix simulation method, being rapid and easy, is clearly useful

Appendix - Tapering Details

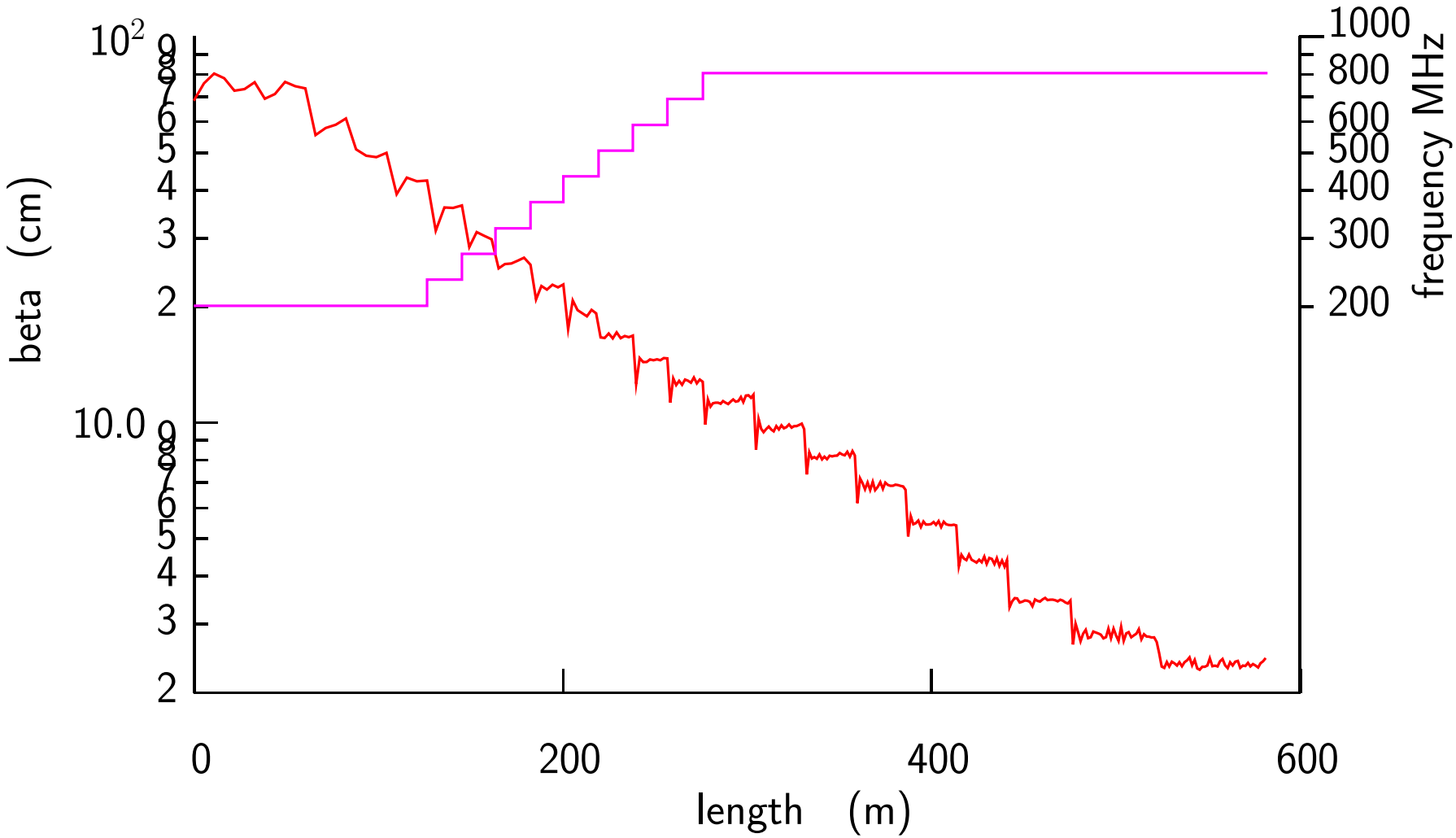
- rf gradients of 15.48 MV/m are assumed for all cases except the 805 MHz stages 16 and higher where it is raised to 18.8 MV/m
- For early tuned cells, hydrogen lengths are fixed
- For scaled cells, hydrogen lengths are proportional to cell lengths
- For scaled cells the rf wavelength is scaled too
  This slow taper of the rf frequency is a temporary assumption
  real frequencies would be 201,402,604,805 MHz with proper matching between them
- When the half hydrogen length is greater than 1.1 times β_o then LiH is used
- Hydrogen window radii are scaled with beta (angular divergence is roughly constant)
- The combined hydrogen and safety windows are taken to be 500 μm for the early 201 MHz stages, and are scaled with their apertures for all subsequent stages

Appendix - Parameters

	β_o cm	file	cells	cell cm	δ	$L_{abs}/2$ cm	R_{abs} cm	Abs	L_{Al} μm	f MHz	L_{rf} cm	G_{rf} MV/m
1	65.6	rfoxb5	12	275	.015	11.3	25	H2	500	201	188	15.48
2	57.6	rfoxb4	10	275	.02	16.3	21	H2	500	201	188	15.48
3	50.0	rfoxb3	18	275	.026	21.3	18	H2	500	201	188	15.48
4	44.7	rfoxb1	18	275	.026	21.3	18	H2	500	201	188	15.48
5	41.0	rfoxb11	18	275	.026	21.3	18	H2	500	201	188	15.48
6	35.0	rfoxb12	8	236	.022	18.2	15	H2	429	235	161	15.48
7	30	rfoxb13	9	202	.019	15.7	13	H2	367	274	138	15.48
8	26	rfoxb14	11	173	.0164	13.4	11	H2	315	319	118	15.48
9	22	rfoxb21	12	148	.014	11.5	9.7	H2	270	373	101	15.48
10	19	rfoxb22	15	127	.012	9.8	8.3	H2	231	435	87	15.48
11	16.2	rfoxb23	17	109	.010	8.45	7.1	H2	198	507	74.4	15.48
12	13.9	rfoxb31	20	93.5	.0088	7.25	6.1	H2	170	592	64	15.48
13	12	rfoxb32	24	80.1	.0076	6.21	5.25	H2	146	690	54.8	15.48
14	10.2	rfoxb33	40	68.75	.0065	5.32	4.5	H2	125	805	4.7	15.48
15	8.5	rbk7a2	40	68.75	.0065	5.32	3.75	H2	105	805	4.7	15.48
16	7.1	rbk8b	40	68.75	.0065	5.32	3.11	H2	87	805	3.84	18.8
17	5.95	rbk8c	40	68.75	.0065	5.32	2.6	H2	72	805	3.84	18.8
18	4.91	rbk8d	40	68.75	.0065	5.32	2.17	H2	60	805	3.84	18.8
19	4.15	rbke2	40	68.75	.006	.96	2.17	LiH	0	805	3.84	18.8
20	3.4	rbkf2	40	68.75	.006	.96	2.17	LiH	0	805	3.84	18.8
21	2.82	rbkg2	40	68.75	.006	.96	2.17	LiH	0	805	3.84	18.8
22	2.37	rbkh2	40	68.75	.006	.96	2.17	LiH	0	805	3.84	18.8

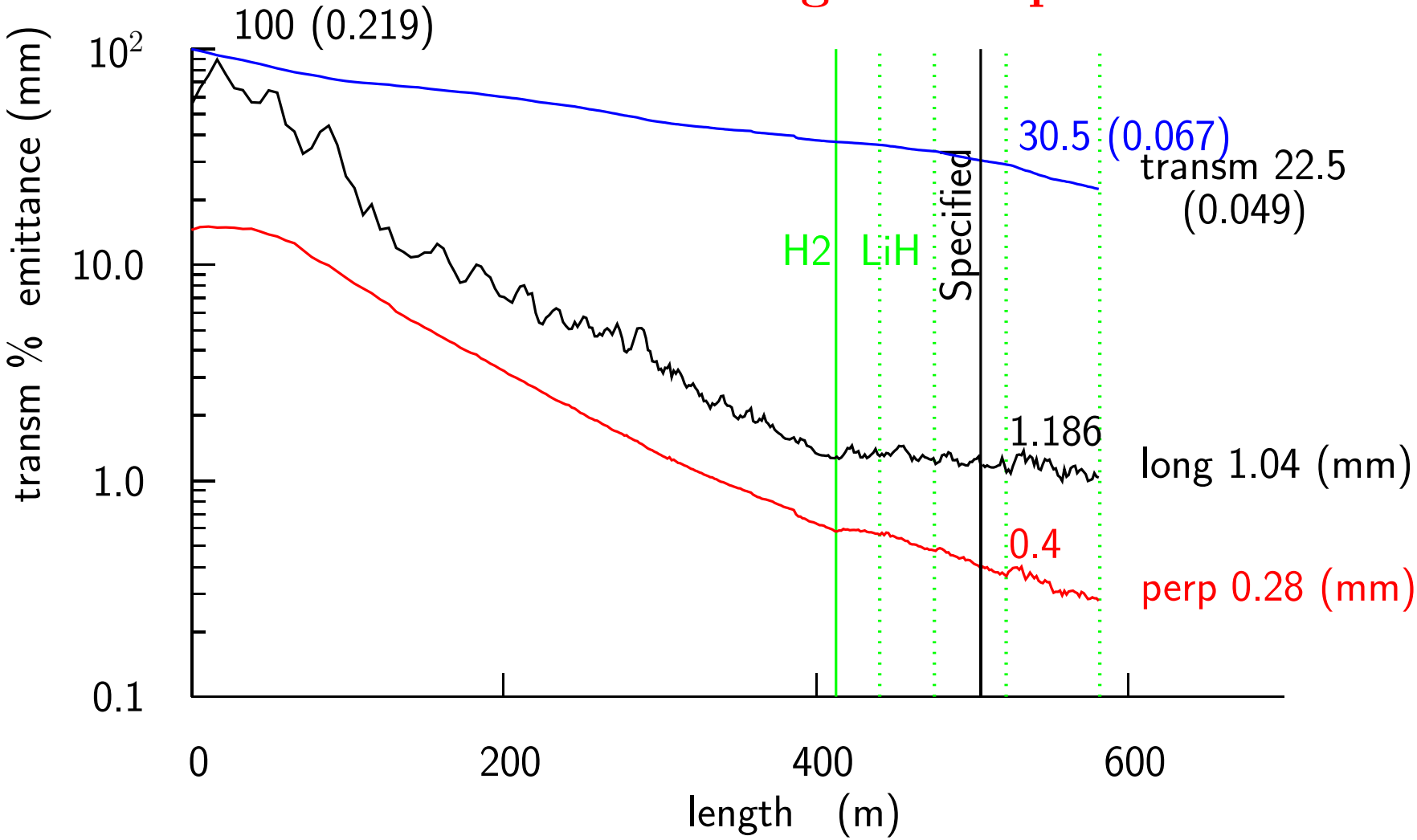
	beta cm	z1 cm	r1 cm	j1 A/mm^2	Bmax1 T	z2 cm	r2 cm	j2 A/mm^2	Bmax2 T
1	65.6								
2	57.6								
3	50.0								
4	44.7								
5	41	15.00 - 35.00	50.00 - 60.00	50.2	4.3	30.00 - 110.00	77.00 - 87.00	55.6	4.3
6	35	12.86 - 30.00	42.86 - 51.43	68.3	5.1	25.72 - 94.29	66.00 - 74.58	75.7	5.1
7	30	11.02 - 25.72	36.74 - 44.09	93.0	5.9	22.05 - 80.84	56.58 - 63.93	103.0	5.8
8	26	9.45 - 22.05	31.50 - 37.80	126.5	6.9	18.90 - 69.30	48.51 - 54.81	140.1	6.7
9	22	7.02 - 19.98	27.00 - 36.18	93.4	7.0	16.20 - 59.40	41.58 - 55.08	86.3	8.0
10	19	6.02 - 17.13	23.15 - 31.01	127.2	8.2	13.89 - 50.92	35.64 - 47.22	117.5	9.5
11	16.2	5.16 - 14.68	19.84 - 26.59	173.1	9.6	11.90 - 43.65	30.55 - 40.47	159.9	10.9
12	13.9	4.42 - 12.59	13.61 - 26.20	102.7	10.3	10.21 - 37.42	26.20 - 46.61	123.0	13.5
13	12.0	3.79 - 10.79	11.66 - 22.45	139.8	12.1	8.75 - 32.08	22.45 - 39.95	167.4	15.7
14	10.2	3.25 - 9.25	10.00 - 19.25	190.2	14	7.50 - 27.50	19.25 - 34.25	227.7	18.4
15	8.5	2.50 - 9.25	9.25 - 19.25	276.0	15.7	10.50 - 28.00	19.25 - 34.25	222.2	17.3
16	7.1	3.25 - 12.50	10.00 - 19.25	217.7	18.0	3.25 - 22.00	19.25 - 34.25	203.1	18.0
17	5.95	3.25 - 12.50	10.00 - 19.25	287.8	20.0	3.25 - 19.50	19.25 - 34.25	191.4	20.0
18	4.91	3.25 - 12.50	7.00 - 21.25	239.7	18.5	13.25 - 23.25	19.25 - 34.25	163.8	12.0
19	4.15	3.25 - 12.50	6.50 - 21.75	259.7	19.4	13.25 - 23.25	19.25 - 29.25	133.2	12.0
20	3.4	3.00 - 13.00	6.50 - 21.75	291.9	20.8	15.50 - 25.50	19.25 - 24.25	0.0	20.8
21	2.82	2.50 - 13.00	4.88 - 19.63	257.5	19.2	15.50 - 25.50	19.25 - 24.25	0.0	19.2
22	2.37	2.50 - 13.00	4.25 - 15.00	294.7	20.0	15.50 - 25.50	19.25 - 24.25	0.0	20.0
11a	10.15	2.50 - 9.25	9.25 - 19.25	193.6	13.2	10.50 - 28.00	19.25 - 34.25	244.4	19.0
18a	2.8	2.50 - 13.00	4.25 - 15.00	263.1	18.7	15.50 - 25.50	19.25 - 24.25	151.5	8.2

Betas vs length



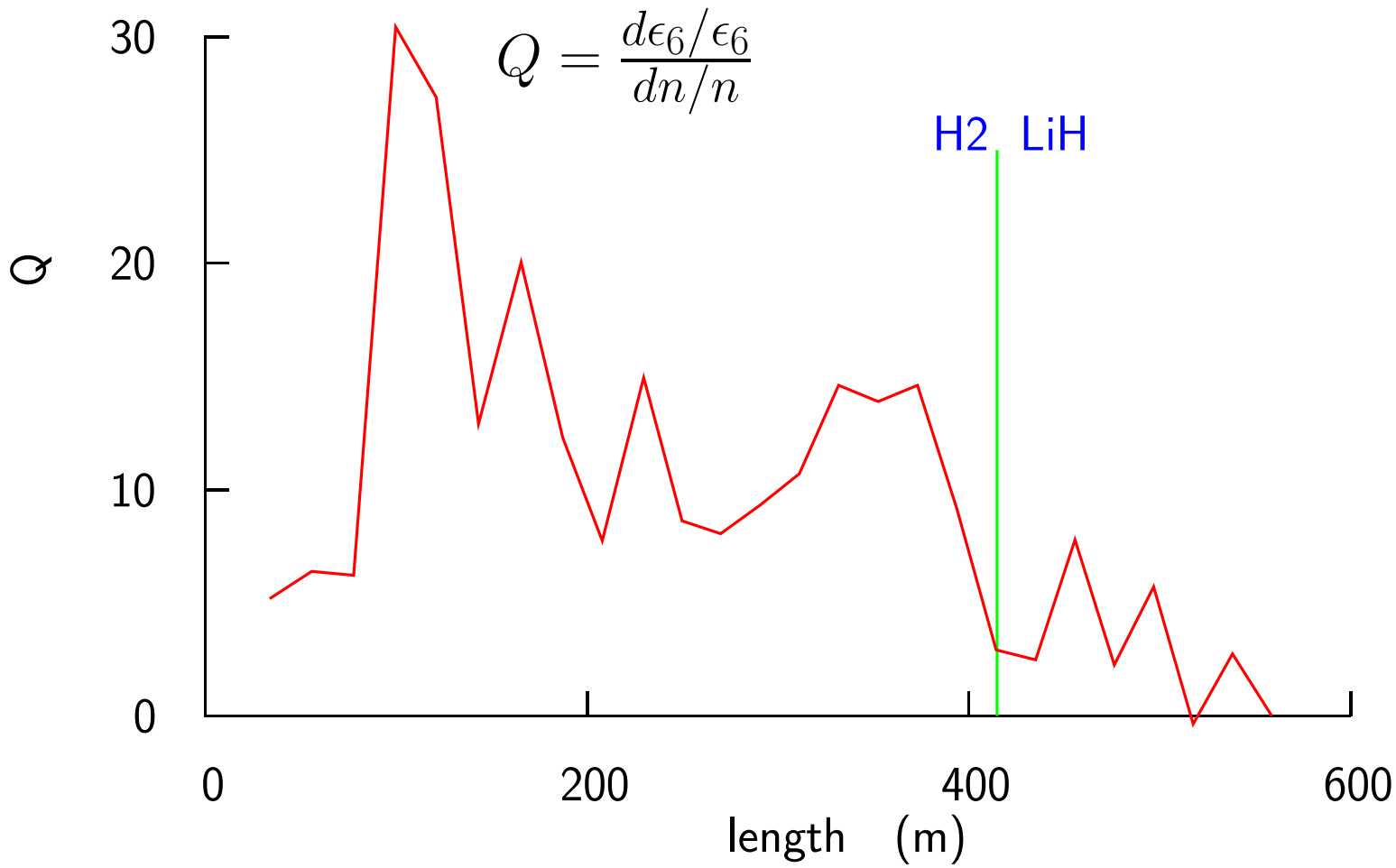
- Beta varied from 80 to 2.5 cm (32:1)
  (the inital observed beta is higher than that on momentum from averaging over momentum spread
- Some beta "bounce" at changes

Simulated emittances vs length in tapered channel



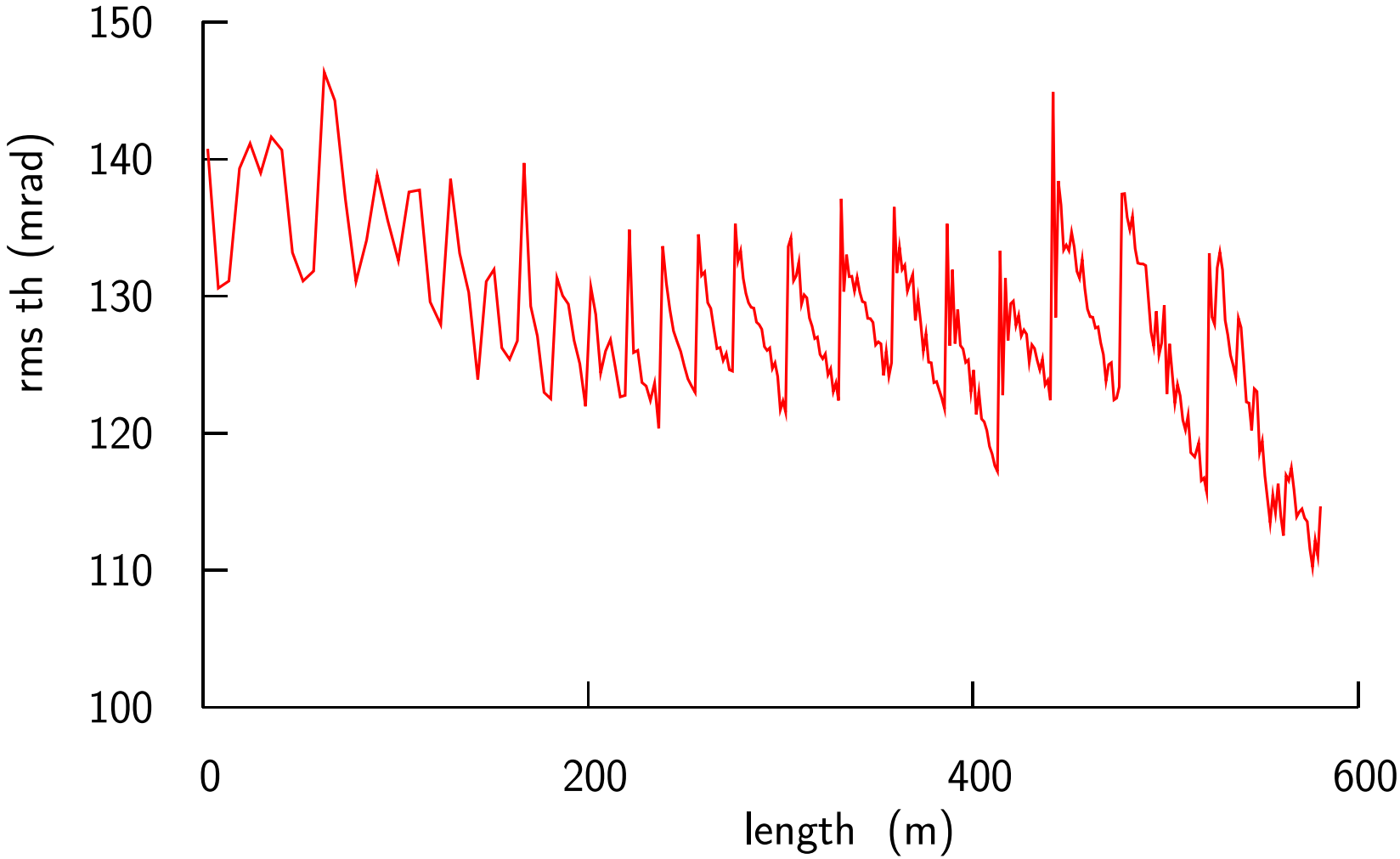
- No transverse emittance growth at matches is observed
- Muons per 8 GeV proton (at $\epsilon_\perp = 400\ \mu m$) = 0.068 (cf ISS 0.08)
- Muons per 8 GeV proton (at $\epsilon_\perp = 260\ \mu m$) = 0.049 (cf ISS 0.08)
- Merit factor = 180,000 x 0.22 = 41,000

Q



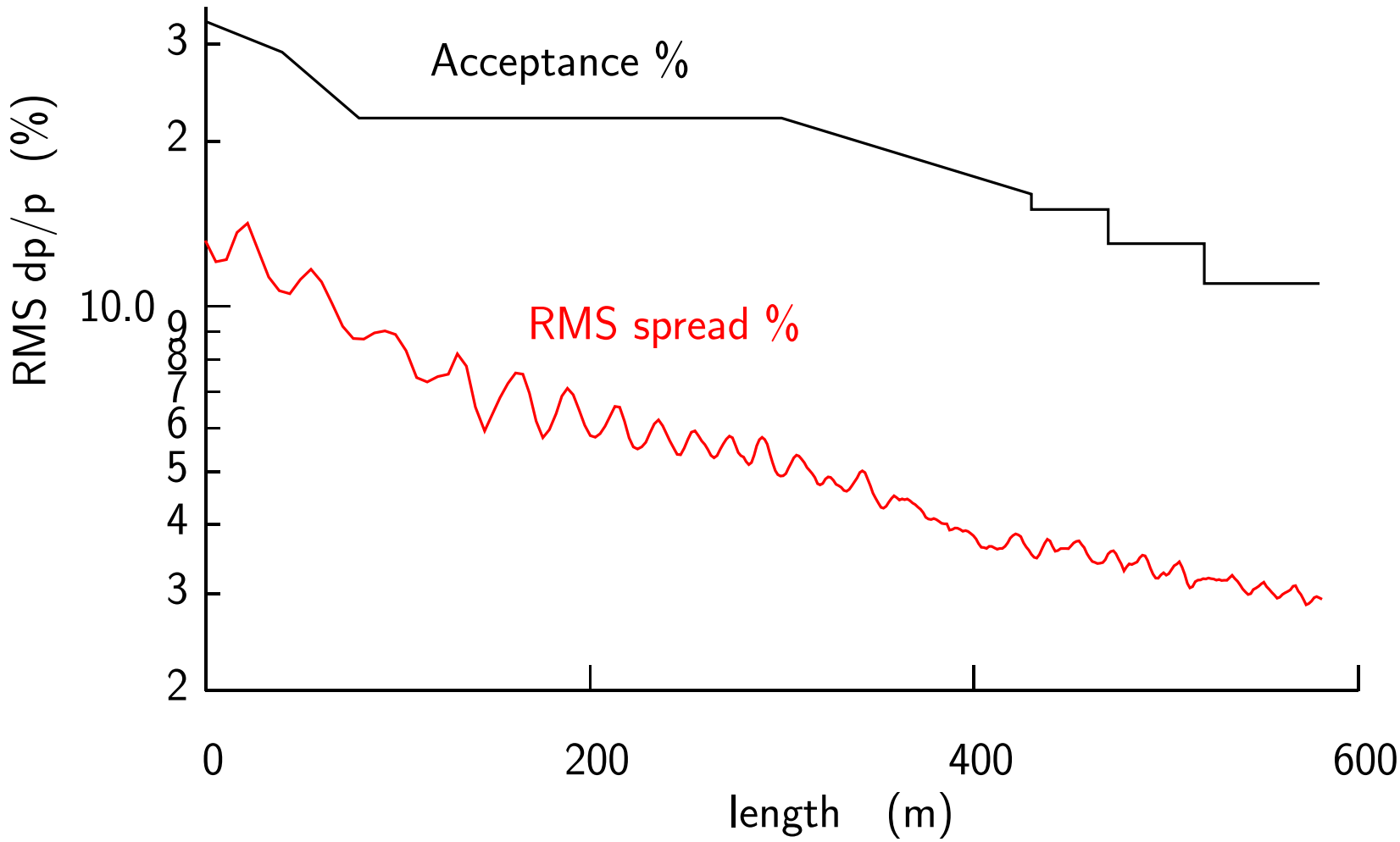
- Maximum Q is similar to un-tapered lattice (≈ 20)
- Final Q in H_2 is similar to un-tapered lattice (≈ 10)
- Final Q in LiH is less (≈ 5)
- Reduction from smaller rad coils and approach to equilib long emittance

rms angles vs length



- Ideal tapering requires constant σ_θ approximately achieved near 130 mrad
- Equilibrium $\approx$ 90 mrad <Cooling Rate> $\approx$(1 - [90/130]^2) $\approx$ 0.5 $\times$ ideal
- Within each stage σ_θ falls, but is restored by following reduction in beta

dp/p vs length



- Equilibrium dp/p≈ 3% approached at end
- Over most of length dp/p >> equilibrium giving efficient longitudinal cooling
- Acceptance > 3 sigma over most of length